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ABSTRACT

This report describes and develops a model for coordinating a variety of demographic and social statistics within a single framework. The framework proposed, together with its associated methods of analysis, serves both general and specific functions. The general aim of these functions is to give numerical definition to the pattern of society and to the way in which this pattern changes over time. Their specific aim is to provide a comprehensive and consistent basis for education and manpower research, policy, and planning. (Tables appended may reproduce poorly.) (Author/RA)

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# demographic accounting and model-building

by

RICHARD STONE

EA 003 876

ORGANISATION FOR ECONOMIC COOPERATION AND DEVELOPMENT
Paris 1971

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*The Organisation for Economic Co-operation and Development (OECD), was set up under a Convention signed in Paris on 14th December, 1960, which provides that the OECD shall promote policies designed :*

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## PREFACE

Almost two decades have passed since national income and product accounts were first developed, and in the course of their evolution Professor Stone has played a most significant role. It was, therefore, fitting that this Organisation, where the initial major effort in the direction of a standardized system of accounts was made, should invite Pofessor Stone to prepare the present report which opens up a new and exciting phase in the development of social accounting.

The report has been developed under the programme of the Committee for Scientific and Technical Personnel as part of the Commitee's broader concern for the promotion of educational planning statistics and methodologies, and its final formulation benefited from intensive discussions of earlier drafts by a group of experts and of educational planners from the Member countries. Within this context, the author has purposefully limited the scope of the report to systematizing information on human stocks and flows in their learning and earning activities. A "system of demographic accounts" emphasizes the in-flow, through-flow and out-flow of humans in the education system, which in turn is imbedded in a wider context linking the population at the beginning of a period with population at the end of the period.

The report is divided into two main parts: Chapters I through VIII, which are concerned with description; and Chapters IX through XI, which are concerned with analysis. Under the name of "demographic accounting" the first part sets out a method of integrating and presenting information on human stocks and flows, and elaborates it with special reference to learning and earning activities. For a variety of purposes, these demographic accounts need to be related to the national economic accounts as presented, for example, in *A System of National Accounts* (Studies in Methods, series F, No. 2, Rev. 3, United Nations, New York, 1968). This link is illustrated in this report by the economic accounts of the educational system, which are set out in Chapter VIII.

Like the economic accounts, the demographic accounts can be regarded simply as a convenient means of organising and arranging statistical data, as a finished product in themselves. Alternatively, they can be regarded as a starting point for analysis. Accordingly, the second part of the report sets out and illustrates a form of input-output model based on the demographic accounts and shows how any relevant information from the economic accounts can be incorporated into demographic model-building. Chapter XI contains a discussion of the problem of allowing for changes through time in the coefficients, a problem that must be solved if the models are to be used in making projections and not solely in analysing the structure of the system as it is at present or as it has developed historically.



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The final chapter sums up the main conclusions of the report and outlines some further developments which lie outside its scope.

The framework proposed by Professor Stone provides a useful basis towards the evolution of an operating accounting system which, in turn, could be field-tested in concrete country situations. As Professor Stone remarks in his conclusions: "... it is already possible to set up a simple system of standard demographic accounts which would enable the position in different countries to be compared without too much delay and at the same time would point the way to constructive developments". Therefore, what is now needed is sufficiently varied empirical experience, even of a tentative nature, in the construction of these accounts for the strengthening of the conceptual foundations elaborated in the report.

In addition, a natural sequel to this work will be in the field of model construction. The suggestion of the Thonstad-Stone type of finite difference models constructed along the lines of the dynamic input-output models of the economy deserves serious consideration. While not precluding alternative approaches to model construction, the input-output framework seems to offer great promise.

It is hoped that the publication of this report will encourage further work in the Member countries towards the development of a fully empirical system of demographic accounts.

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*Part One*

# DESCRIPTION

## DEMOGRAPHIC ACCOUNTING AND ITS LINKS WITH THE ECONOMIC ACCOUNTS

# I

# WHAT IS DEMOGRAPHIC ACCOUNTING ?

## 1. Scope and Aims

This report describes a means of co-ordinating within a single framework a great variety of demographic and social statistics, shows how this information can be used analytically and sets out its relationship to the entries in the national economic accounts. The framework proposed, a system of demographic accounts, as it will be called, and the methods of analysis associated with it, are designed to serve two aims, one general and one specific. The general aim is to give numerical definition, in so far as this is possible, to the pattern of society and to the way in which this pattern changes through time. The specific aim is to provide a comprehensive and consistent basis for education and manpower research, policy and planning.

A system of demographic and social statistics such as that presented here makes it possible to integrate information on human stocks and flows just as a system of national accounts statistics makes it possible to integrate information on economic and financial stocks and flows. The main difference between the two cases is that while a generation of endeavour and experience lies behind the current developments of national economic accounting, a generally agreed system has not yet been formulated in the social and demographic field. Information exists, of course, and has long existed about many of the characteristics of human beings: their age and sex, their marital status, their family and political attachments, their medical and criminal records, their education, their niche in economic life and so on. But this information is not integrated and does not form a system as does information on production, consumption, accumulation, foreign trade, prices, taxes, saving, capital, labour and so on when organised within the framework of the national accounts. The reasons for attempting integration, however, are as strong in the one case as in the other.

In the first place, an integrated system helps to indentify the types of analysis that seem desirable and the data required for these analyses. For example, when making educational projections it is necessary to take into account, among other things, current changes in the educational system. Educational statistics, though often voluminous, usually concentrate much more heavily on stocks than on flows, with the consequence that they are not well-suited to an analysis of the mechanism of change. For this type of analysis we need data on flows; and such data may be of different kinds. In the simplest case, information is provided on year-to-year links between

the different categories of the system, and this implies that the destinations of the outflows are independent of the sources of the inflows. A more complicated case arises when we want to relate outflows and inflows; in general, this requires much more detailed information and the possibility of linking different kinds of records. The more comprehensive and coherent our frame of reference, the easier will it be to make the right connections.

In the second place, an integrated system helps to check the consistency of the data to be used. This is particularly important when we want to combine data from what are conventionally regarded as separate statistical universes. For example, the demand for a place in a university may depend on a complex of factors, such as type of school, personal ability, family background and home locality, all of which contribute to determine the attractiveness of and opportunity for higher education. In this case we need information on applications for admission cross-classified by these criteria, some of which lie outside the purlieu of educational statistics as they are understood at present. Similarly, for many purposes it is necessary to link flows out of the educational system with flows into employment. This means combining educational data with employment data. The attempt to co-ordinate these different branches of statistics brings many gaps and inconsistencies to light.

In the third place, an integrated system helps to find out how far existing data can be adapted to analytical needs and how far new data are desirable. Data cost money and methods of collection cannot be changed overnight, so that adapting these methods to meet changing needs gives rise to considerable practical difficulties.

In the fourth place, an integrated system helps to discriminate in the production of statistics and to establish an order of priorities in the collection of new information. Data are only useful if they are used, and a mass of unusable or rarely used data creates almost as serious a problem as does a lack of data that would be useful.

Finally, an integrated system is indispensable for computing purposes. The efficient operation of high-speed computers demands an exact understanding of the way in which innumerable bits of information are connected and sets a premium on a coherent framework in which every bit is seen to have its place.

Of the many facets of life which characterize a social system, only learning and earning activities are considered in any detail in this report. Even in its restricted sphere of application, the report does no more than make a start. But as the development of the national economic accounts has shown, this is the only practical way to proceed. What is wanted at the beginning of any such venture is a simple and consistent framework which connects the most important variables to be studied and which is capable of elaboration as information and experience accumulate.

## 2. Substantive Uses

Demographic accounting, as has been said, provides a quantitative description of certain aspects of society. Its substantive uses lie in those areas of policy-making and planning where such information is needed. The accounts discussed in this report describe the flows of human beings through learning and earning activities. Consequently their substantive uses are to be found in educational planning, manpower planning and

economic planning in general. A number of possible uses can be identified as follows.

a) *In educational planning*

First, to provide a consistent body of information on the numbers of pupils found in the various stages of the educational system at the beginning and end of a given period and on the numbers flowing into, through and out of the system during that period. These data afford a quantitative description of the structure of the educational system, of the way in which its different parts are connected. The compilation of such data for a sequence of periods shows how the structure of the system is changing.

Second, to make projections based in part on demographic changes and in part on the continuation of changes in the structure of education as they appear to be taking place at present or as they might be expected to take place if certain policies were to be adopted.

Third, to make projections of educational costs by linking educational activity levels to input and cost structures in various branches of the system.

Fourth, to indicate the prior provisions, particularly in teacher training and in the construction of educational buildings, needed to realize a given educational programme.

Fifth and general, to provide data over and above the examples just given which will help in the efficient management of the educational system as a whole.

b) *In manpower planning*

First, to provide a consistent body of information on the changing age-composition of the labour force in different industries, on the educational qualifications of new entrants and on the qualifications at least of those engaged in certain occupations.

Second, to contribute towards the identification of potential imbalances in the future demand for and supply of certain kinds of labour, thus helping in the assessment of educational and other policies designed to diminish these imbalances.

Third, to provide basic data for decision-making by individuals, educational authorities, professional associations and employers on such matters as the choice of a career, the structure of education and the policy of recruitment.

c) *In economic planning generally*

First, to trace the connections between education and employment and thus contribute to the formulation of consistent educational and manpower policies.

Second, to provide the data needed for discussing the allocation of economic resources, particularly in the field of investment in education.

Third, to contribute to the data needed for regional planning. The regional aspect of demographic accounting is not emphasized in this report but its importance is evident, since regional planning usually starts from a demographic base.

# II

# SETTING UP A SYSTEM OF DEMOGRAPHIC ACCOUNTS

## 1. Outlines of the System

Demographic accounts, like any other type of account, are based on the equality of inflows and outflows over a period of time. In the demographic case these inflows and outflows are human beings who, in a given period, enter or leave a country. These human flows move in two dimensions, time and space. Thus, for any country, this year's population flows in partly along time from last year, through survival, and partly along space from the outside world, through birth and immigration; and flows out partly along space into the outside world, through death and emigration, and partly along time into next year, through survival.

The survivors from the preceding period constitute the opening stock and the survivors into the succeeding period constitute the closing stock. In any period, the equality of inflows and outflows implies that the difference between the opening and the closing stock is equal to the net inflow from the outside world.

The first step in the construction of a system of demographic accounts is to divide the population into age-groups and, within each age-group, into categories reflecting any characteristic we may be interested in: activity, socio-economic position, health and so on. Once the data have been organised within a taxonomic framework, they can be presented as entries in an account showing, on one side, in what formation each age-group enters the period and, on the other side, how it redistributes itself among the various categories before leaving the period. The accounting identities connecting flows and stocks will ensure that the same number of people will flow out of a category as have flowed into it.

Before setting up the accounts, however, it is necessary to have a clear idea of how one intends to define one's terms. What is the population of a country? How long should the period of account be? How is age to be recorded? What are the principal characteristics that ought to be brought out and how should they be combined? Which is the best way of presenting the information? These and other such basic questions are discussed in the following sections.

## 2. National and Domestic Concepts

This distinction, familiar to national accountants, applies equally in demographic accounting. We may decide to keep accounts either for the

normal residents of a country (the national concept) or for the actual residents (the domestic concept). Normal residents are people who consider the country as their home country; actual residents are people who happen to be living in it at a particular time.

The relevance of the distinction can be seen from an example. If we are interested in the educational development of our country's students, we shall be concerned with the flow of these students both through our universities and through those of foreign countries. This is the national concept. If, on the other hand, we are interested in the administrative problems of our country's universities, we shall be concerned with the flow of students, whether domestic or foreign, through our universities. This is the domestic concept.

### 3. Migrants and Visitors

In order to give effect to the distinction between national and domestic population, it is necessary to classify individuals moving between our country and the rest of the world into migrants and visitors. Migrants are individuals whose displacement is intended to be permanent and who, therefore, become normal residents of the country into which they move. Visitors, on the other hand, are individuals whose displacement is intended to be temporary and who remain, therefore, normal residents of the country from which they are temporarily absent.

These definitions make use of the criterion of intention, and it must be recognized that visitors may end by becoming migrants and that migrants may change their mind and go back to their country of origin, or migrate elsewhere, after a short time. It is also true that some people, such as business representatives abroad and colonial civil servants, have strong ties with more than one country. Finally, there are various types of migrant labour whose country of residence for the time being is dictated by the state of the labour market. These workers may eventually settle down in a country other than their own but, until they do, they are perhaps best regarded as visitors.

Detailed solutions are required for problems such as these. Whatever form these solutions may take, it can normally be expected that migrants will constitute a relatively small category and visitors a relatively large one. The whole problem is likely to increase in importance the smaller the territorial area considered and the stronger the economic and social links between this area and its neighbours.

### 4. Time Intervals

As we have seen, demographic accounts link the opening stock of population at the beginning of a period (the survivors from the preceding period) with the closing stock at the end of the period (the survivors into the following period). As with economic accounts, the time interval will normally be a year, although in principle there is no difficulty in setting up accounts for shorter periods.

If the year is accepted as the time interval, there remains the question of the starting date. If 1st January is chosen, the stocks will relate to the beginning and end of calendar years and the flows will be those into and out of calendar years. Since the national economic accounts are usually

kept for calendar years, there might be some advantage in making 1st July the starting point for the demographic accounts, since this would automatically provide a mid-year estimate of the labour force. From an educational point of view, on the other hand, a case could be made for a year beginning on 1st October, say, since this would provide information on the distribution of students at the beginning of the academic year. Whatever decision is made, it is important that the same date should be chosen for all demographic statistics.

As often happens, the initial choice is likely to be strongly influenced if not actually determined by the statistics available. Only as the collection of more statistics comes to be planned, will it usually be possible to revise the initial choice. In the meantime, experience will have been gained which will make the revision easier.

The choice of time interval affects the treatment that it is reasonable to apply to the recording of visitors. Generally speaking, we are not much interested in recording visitors who enter and leave within the chosen interval. Thus the vast majority of visitors, largely composed of tourists, whose average stay in a foreign country is probably about a fortnight would be excluded from annual accounts; only those visitors who happen to be present at the beginning or at the end of the period would be recorded. Indeed, for many purposes the omission of repeated inward and outward movements of short duration is essential in order to avoid duplication, as the following example shows.

Border workers, who live in one country and work in another, move almost daily between the two countries. They are normal residents of the country in which they live and form part of the domestic labour force of the country in which they work. If the accounts are kept on a domestic basis for the country in which they work, these workers will appear both as an inward and as an outward movement associated with the industry in which they work. And the labour force of that industry, calculated as the sum of normal residents and border workers, will be correctly estimated only if border workers who work throughout the year are shown as making a single inward and outward movement. Of course, if the accounts were kept by days rather than by years every inward and outward journey would appear automatically. This is a good illustration of how the movements to be recorded are dependent on the time interval for which the accounts are kept.

## 5. The Definition of Age

It is convenient to define age as age last birthday at the end of the period of account, which in this report is assumed to be the calendar year. This means, for instance, that all those who are born in 1960 are aged 0 at the end of that year, have their first birthday in 1961, are aged 1 at the end of that year, and so on.

Having adopted a definition of age, all the information used must be adjusted to it. Much of this information is expressed in terms of the age at which something happens: an individual leaves school, or enters employment, or dies. All such information must be adjusted to the age which the individual reached or, in the case of death, would have reached, at the end of the calendar year in which the event took place. For example, the deaths to be recorded under age 0 in 1960 are the deaths of those born

in 1960. Statistics of deaths usually show, instead, the deaths in 1960 of children aged 0 at time of death. But some of these children will have been born in 1959, and so their death in 1960 should be recorded under age 1. In a similar way, some of the 1-year-old deaths have to be transferred to age 2, and so on.

Adjustments of this kind are not only time-consuming but are also a source of inaccuracy in demographic accounting, since the proper basis for the adjustment is often not as clear, and the basic data are often not as complete, as in the example just given. As a consequence, it is desirable to establish a tabulation based on a standard definition of age wherever this is possible.

## 6. Categories and States

A system connects a number of categories of objects, and the elements of each category must be grouped according to a classification which is both internally consistent and capable of interacting with the elements of other categories. The categories and classifications appropriate to a system of demographic accounts are partly institutional, partly familial and partly personal. Examples of the first are the type of school which a pupil attends, the industry to which an employee is attached, the hospital where a patient is treated. Examples of the second are family size, social class, income group. Examples of the third are age, sex, subject of study, educational qualifications, occupation, health and criminal records.

The interaction of different classifications forms a number of composite categories, or "states", which enable us to subdivide the population into clearly-differentiated groups. Thus if our primary classification is by age, our secondary classification by centre of activity and our third classification by occupation, examples of states are: being aged 17, attending grammar school and preparing for advanced-level examinations; being aged 30 and working in a brick factory as a clerk.

## 7. Sequences

In a system as complex as the one we are dealing with, even a relatively modest degree of cross-classification can lead to an immense amount of statistical detail which will be very difficult to handle unless we find some way of keeping it under control. This can be achieved by devising a comparatively simple set of basic data, which might be thought of as the master set, supported by a number of subsidiary sets linked to each other and to the master set by the conceptual framework. These subsidiary sets can take the form of "life sequences", each sequence reflecting a particular dimension of life and requiring only a limited number of classifications for its description. At the same time, each sequence must be so conceived as to be demographically self-contained, that is, to be capable of embracing the whole population.

For example, this report is mainly concerned with education and employment considered as part of an "active" sequence, in which people first learn, then earn and finally retire. In this sequence the population can be divided into three main groups: pupils and students, members of the labour force, and the rest. Some of the classifications appropriate to the first group are: type of educational establishment attended, subjects studied,

qualifications obtained. Some of the classifications appropriate to the second group are: industry of employment, type of occupation, level reached in that occupation. The third group contains simply the educationally and economically inactive. Its main components are: babies under school age; children of school age who for one reason or another do not attend school; the important category of housewives; the unoccupied of both sexes who dislike work and manage to resist social pressures in its favour, namely the well-to-do who pass their time in cultural or sporting pursuits and tramps and destitutes who somehow muddle through; the long-term inmates of hospitals and prisons, namely the chronically sick and mentally deranged and criminals serving long prison sentences; and, finally, the retired. None of these people, except possibly the very young, the very sick and the very old, can be said to be inactive; but their activities escape clear-cut definition and so have to be treated residually.

While flowing through their active sequence, individuals also flow through a "passive" sequence, in the sense of a succession of socio-economic groupings to which they belong at different times of their life; some of the classifications appropriate to this sequence are type of household (natural or institutional), family size, social class, income group, place of residence, race, religion. Another important sequence is the medical sequence, in which the population is classified in terms of physical and mental characteristics. Yet another is the sequence of deliquency, which can range from parking offences to murder.

Having found a practical means of representing each sequence separately, in then becomes possible to introduce a certain amount of interaction between the different sequences. How we combine the classifications will depend on which aspect of the social scene we are interested in. For example, at the educational stage, family backgroung is an important determinant of educational aspirations and educational achievement; at this stage of the active sequence, therefore, it is desirable that some of the classifications of the passive sequence should interact with those of the active sequence. Again, intelligence, mental balance, physical fitness all have a bearing on people's performance in their work, and so we may want to introduce some interaction between the active sequence and the medical sequence.

In this report only the learning and earning stages of the active sequence are considered in detail; they are discussed in Chapter V and VI respectively. Chapter VII contains a brief outline of the passive sequence.

### 8. Cross-Sections and Vintages

In economics it is customary to distinguish between cross-section analysis and time-series analysis. In demography we can make a similar distinction: we can consider the changes of state taking place in a particular year of all the people who flow into that year; or we can trace through a succession of years the changes of state of the people born in one year.

In this report we shall be concerned almost exclusively with cross-section analyses. But similar methods can be applied to analysing the progress through life of a single cohort or, to use a more appropriate metaphor, the maturing of a single vintage. Naturally, the analysis of any vintage born less than a lifetime ago cannot be complete. This will become apparent from the numerical examples given in section 9 of the next chapter.

## 9. Methods of Presentation

However records are kept, whether on worksheets, on punched cards, on magnetic tape or in the disc store of a computer, there are various ways in which they can be presented on the printed page. Any of these ways is mainly of interest for two purposes: *i*), in numerical form, to promote familiarity with the structure of the system by enabling the reader to see for himself how its elements are related and to check that its arithmetical properties are satisfied; and *ii*), in symbolic form, to forward the task of computerizing the system by providing a visual index to the position of its elements.

Among users of complex statistical systems, some are interested in the system as a whole: what these people want is a bird's eye-view of the data. Others, the majority, have little concern with the system as such: what they want are close-ups of those parts they are interested in, extracted from the system either by simple tabulation or by more sophisticated analytical means. In either case, they will need a presentation of the data which is at the same time clear, consistent and self-contained.

Three methods of presentation which satisfy these conditions will be described in this report:

*a*) accounts showing the balance of inflows and outflows in a period;
*b*) matrices connecting the opening and closing stocks of a period; and
*c*) matrices connecting the outflows of a period with the inflows into the succeeding period.

Any of these methods can be used to set up accounts for the whole population, for vintages, for single ages or age-groups, for separate activities, or indeed for any category into which we may wish to divide the population. All three methods will be illustrated in the next chapter, but that chosen for detailed development in later chapters is the third.



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# III

## A NUMERICAL EXAMPLE

### 1. Alternative Approaches

From what has been said so far, it is obvious that a system of demographic accounts can be set up in many different ways, all mutually consistent. In theory, which way we choose depends simply on which aspect of the system we want to emphasize. In practice it may also depend on the data at our disposal; but this type of constraint will be discussed fully in the next chapter.

In this chapter, the comparative merits of alternative approaches are illustrated by a numerical example relating to the male population of England and Wales in 1965. With the data available at present, some of the entries, in particular those concerning movements to and from the rest of the world, are no more than indications of orders of magnitude. But in an example this does not much matter, especially as the information about flows within the country is fairly accurate.

### 2. National and Domestic Accounts

Before going any further, let us look at the information needed to set up demographic accounts on either a national or a domestic basis. It is convenient to divide this information into three groups:

*a*) normal residents of our country who are in the country at the beginning or end of the period of account, or who enter or leave the country in the course of the period;

*b*) normal residents of our country who happen to be abroad at the beginning or end of the period, or who move to or from the rest of the world in the course of the period;

*c*) foreign visitors who happen to be in our country at the beginning or end of the period, or who enter or leave our country in the course of the period.

The information thus classified can be set out as in Table III.1.

If we consolidate accounts (*a*) and (*b*) of Table III.1, we obtain a demographic account for normal residents. This is set out in Table III.2.

This account corresponds to the national concept: it is an account for normal residents wherever they happen to be. Thus in Table III.2 there were 23,254,600 normal residents of England and Wales at the beginning of 1965, of whom 134,000 were abroad at the time. This opening stock was increased in the course of 1965 by 443,200 births and 144,300

TABLE III.1. BASIC DATA FOR THE CONSTRUCTION OF DEMOGRAPHIC ACCOUNTS, ENGLAND AND WALES, MALE POPULATION, 1965

*Thousands.*

| Inflows | | Outflows | |
|---|---|---|---|
| *a)* NORMAL RESIDENTS OF ENGLAND AND WALES FLOWING INTO AND OUT OF 1965. *(In England and Wales).* | | | |
| Survivors from 1964 | 23,120.6 | Deaths in 1965 | 282.1 |
| Births in 1965 | 443.2 | Emigrants in 1965 | 128.3 |
| Immigrants in 1965 | 144.3 | Visitors to the rest of the world leaving in 1965 | 8.7 |
| Visitors to the rest of the world returning in 1965 | 8.4 | Survivors to 1966 | 23,297.4 |
| Total | 23,716.5 | Total | 23,716.5 |
| *b)* NORMAL RESIDENTS OF ENGLAND AND WALES FLOWING INTO AND OUT OF 1965. *(In the rest of the world).* | | | |
| Visitors from England and Wales surviving from 1964 | 134.0 | Deaths of visitors from England and Wales in 1965 | 0.5 |
| Visitors from England and Wales entering in 1965 | 8.7 | Visitors from England and Wales leaving in 1965 | 8.4 |
| | | Visitors from England and Wales surviving to 1966 | 133.8 |
| Total | 142.7 | Total | 142.7 |
| *c)* FOREIGN VISITORS TO ENGLAND AND WALES FLOWING INTO AND OUT OF 1965. *(In England and Wales).* | | | |
| Foreign visitors surviving from 1964 | 20.0 | Deaths of foreign visitors in 1965 | 0.2 |
| Foreign visitors entering in 1965 | 15.5 | Foreign visitors leaving in 1965 | 15.2 |
| | | Foreign visitors surviving to 1966 | 20.1 |
| Total | 35.5 | Total | 35.5 |

TABLE III.2. A DEMOGRAPHIC ACCOUNT FOR NORMAL RESIDENTS ENGLAND AND WALES, MALE POPULATION, 1965

*Thousands.*

| Inflows | | | Outflows | | |
|---|---|---|---|---|---|
| Survivors from 1964: | | | Deaths in 1965: | | |
| at home | 23,120.6 | | at home | 282.1 | |
| abroad | 134.0 | | abroad | 0.5 | |
| | | 23,254.6 | | | 282.6 |
| Births in 1965 | | 443.2 | Emigrants in 1965 | | 128.3 |
| Immigrants in 1965 | | 144.3 | Survivors to 1966: | | |
| | | | at home | 23,297.4 | |
| | | | abroad | 133.8 | |
| | | | | | 23,431.2 |
| Total | | 23,842.1 | Total | | 23,842.1 |

immigrations; and was diminished by 282,600 deaths and 128,300 emigrations. As a result, the closing stock was 23,431,200, an increase of 176,600 over the opening stock, made up of an excess of births over deaths of 160,600 and an excess of immigrants over emigrants of 16,000.

If we combine accounts (*a*) and (*c*) of Table III.1, we obtain a demographic account for actual residents. This is set out in Table III.3.

TABLE III.3. A DEMOGRAPHIC ACCOUNT FOR ACTUAL RESIDENTS
ENGLAND AND WALES, MALE POPULATION, 1965

*Thousands.*

| Inflows | | | Outflows | | |
|---|---|---|---|---|---|
| Survivors from 1964: | | | Deaths in 1965: | | |
| normal residents.. | 23,120.6 | | normal residents .... | 282.1 | |
| foreign visitors .... | 20.0 | | foreign visitors ..... | 0.2 | |
| | | 23,140.6 | | | 282.3 |
| Births in 1965 ..... | | 443.2 | Emigrants in 1965 ..... | | 128.3 |
| Immigrants in 1965 ..... | | 144.3 | Foreign visitors leaving in 1965.. | | 15.2 |
| Foreign visitors entering in 1965.. | | 15.5 | Visitors to the rest of the world leaving in 1965 ..... | | 8.7 |
| Visitors to the rest of the world returning in 1965 ..... | | 8.4 | Survivors to 1966: | | |
| | | | normal residents.. | 23,297.4 | |
| | | | foreign visitors .... | 20.1 | |
| | | | | | 23,317.5 |
| Total ..... | | 23,752.0 | Total ..... | | 23,752.0 |

This account corresponds to the domestic concept: it is an account for actual residents wherever they may come from. Thus in Table III.3 there were 23,140,600 actual residents in England and Wales at the beginning of 1965. This is less than the figure for normal residents because at the time there were fewer foreigners in the country than there were normal residents of England and Wales abroad. The opening and closing stocks are, in this case, connected not only by births, deaths and migration but also by movements of visitors: foreigners visiting England and Wales and foreigners returning abroad; and Englishmen and Welshmen visiting foreign countries and returning to England and Wales. These figures for movements of visitors may appear very small in as much as in 1965 the total flow of foreign (male) visitors to this country was of the order of 1,800,000 and the total flow of English and Welsh (male) travellers abroad was of the order of 3,200,000. The reason is that the figures in Table III.3 are intended to exclude all short-term visits taking place within the calendar year. Given the approximate character of the figures available for visitors, it has been assumed that the relevant categories of both domestic and foreign short-term travellers increase through time by the same absolute amount, with the consequence that the total net movement of visitors is zero: $(15.5+8.4)-(15.2+8.7) = 0$. This is simply an assumption of convenience which makes it possible to drop these poorly documented items.

## 3. Elaborating the Accounts

Whether we choose to elaborate the accounts on a national or on a domestic basis depends on whether we are more interested in the activities of our own normal residents, both at home and abroad, or in the activities of the inhabitants of our country, be they normal residents or temporary visitors. The development illustrated in this paper is based on the domestic concept.

Unfortunately it is impossible in this development to preserve the full recording of migrations because there are no statistics of the age distribution of immigrants and emigrants. Thus when it comes to subdividing the population by age, the only way of dealing with migration is to treat it at each age as a residual, that is as the difference between the number of people aged $\theta$, say, at the beginning of the year and the number aged $\theta+1$ at the end the year, making allowance for births and deaths. This difference is entered as "net immigration", and the flows to the outside world consist only of deaths.

With this proviso in mind, the starting point of our development, derived from Table III.3, can be set out as in Table III.4.

Table III.4. A DEMOGRAPHIC ACCOUNT FOR ACTUAL RESIDENTS WITH MIGRANTS AND VISITORS SHOWN NET
England and Wales, male population, 1965

*Thousands.*

| Inflows | | Outflows | |
|---|---|---|---|
| Survivors from 1964 | 23,140.6 | Deaths in 1965 | 282.3 |
| Births in 1965 | 443.2 | Survivors to 1966 | 23.317.5 |
| Net immigrants in 1965 | 16.0 | | |
| Total | 23,599.8 | Total | 23,599.8 |

It will be noticed that in all the tables given so far, the entries are shown to a larger number of decimal places than is usually warranted. The reason is that when it comes to subdividing these entries by state, some small but accurate components will make their appearance, as also happens in the economic national accounts, and it is arithmetically convenient to work to a number of decimal places that will allow the small items to be adequately represented. For example, each year about 1,500 2-year-old boys go to nursery school. By taking 1,000 as the unit and working to the nearest 100, we can express this number as 1.5; had we worked to the nearest 1,000, we should have had to express it as either 1 or 2; and had we worked to the nearest 10,000 it would have disappeared altogether.

## 4. A Classification by Age

The entries in Table III.4 can be classified in various ways; for example, by age or by activity. The classification by age can run in terms either of single ages or of age-groups. By way of example, let us classify the pre-school ages, from 0 through 4, by years of age and let us divide the remaining ages into three groups: the ages of compulsory school attendance,

TABLE III.5. DEMOGRAPHIC ACCOUNTS FOR ACTUAL RESIDENTS CLASSIFIED BY AGE

ENGLAND AND WALES, MALE POPULATION, 1965

*Thousands.*

| Inflows | | Outflows | |
|---|---|---|---|
| **AGE 0** | | | |
| Births in 1965 | 443.2 | Deaths in 1965 | 8.3 |
| Net immigrants in 1965 | 0.0 | Survivors to 1966 | 434.9 |
| Total | 443.2 | Total | 443.2 |
| **AGE 1** | | | |
| Survivors from 1964 | 442.2 | Deaths in 1965 | 1.5 |
| Net immigrants in 1965 | 0.0 | Survivors to 1966 | 440.7 |
| Total | 442.2 | Total | 442.2 |
| **AGE 2** | | | |
| Survivors from 1964 | 428.0 | Deaths in 1965 | 0.5 |
| Net immigrants in 1965 | 0.5 | Survivors to 1966 | 428.0 |
| Total | 428.5 | Total | 428.5 |
| **AGE 3** | | | |
| Survivors from 1964 | 420.0 | Deaths in 1965 | 0.3 |
| Net immigrants in 1965 | 0.3 | Survivors to 1966 | 420.0 |
| Total | 420.3 | Total | 420.3 |
| **AGE 4** | | | |
| Survivors from 1964 | 407.0 | Deaths in 1965 | 0.3 |
| Net immigrants in 1965 | −0.7 | Survivors to 1966 | 406.0 |
| Total | 406.3 | Total | 406.3 |
| **AGES 5 THROUGH 15** | | | |
| Survivors from 1964 | 3,863.9 | Deaths in 1965 | 1.8 |
| Net immigrants in 1965 | −3.9 | Survivors to 1966 | 3,858.2 |
| Total | 3,860.0 | Total | 3,860.0 |
| **AGES 16 THROUGH 19** | | | |
| Survivors from 1964 | 1,564.3 | Deaths in 1965 | 1.6 |
| Net immigrants in 1965 | −2.1 | Survivors to 1966 | 1,560.6 |
| Total | 1,562.2 | Total | 1,562.2 |
| **AGES 20 +** | | | |
| Survivors from 1964 | 16,015.2 | Deaths in 1965 | 268.0 |
| Net immigrants in 1965 | 21.9 | Survivors to 1966 | 15,769.1 |
| Total | 16,037.1 | Total | 16,037.1 |

5 through 15; the later school ages, 16 through 19; and the post-school ages, 20+. This detail is set out in Table III.5. As was said in the preceding chapter, age is defined throughout this paper as age last birthday at the end of the year.

If the entries in the separate accounts of Table III.5 are added together, the result is simply Table III.4.

## 5. A Classification by Activity

As an example of a classification by activity, let us consider the dichotomy "at school" and "not at school". This detail is set out in Table III.6.

Table III.6. DEMOGRAPHIC ACCOUNTS FOR ACTUAL RESIDENTS CLASSIFIED BY ACTIVITY
England and Wales, male population, 1965

*Thousands.*

| Inflows | | | Outflows | |
|---|---|---|---|---|
| **At school** | | | | |
| Survivors from 1964: | | | Deaths in 1965 | 1.6 |
| already at school | 3,650.7 | | | |
| entering school in 1965 | 396.1 | | | |
| | | 4,046.8 | Survivors to 1966 | 4,039.9 |
| Net immigrants in 1965 | | −5.3 | | |
| Total | | 4,041.5 | Total | 4,041.5 |
| **Not at school** | | | | |
| Survivors from 1964: | | | Deaths in 1965 | 280.7 |
| not at school | 18,748.8 | | | |
| leaving school in 1965 | 345.0 | | Survivors to 1966 | 19,277.6 |
| | | 19,093.8 | | |
| Births in 1965 | | 443.2 | | |
| Net immigrants in 1965 | | 21.3 | | |
| Total | | 19,558.3 | Total | 19,558.3 |

Again, if the entries in the separate accounts of Table III.6 are added together, the result is Table III.4.

## 6. A Classification by Age and Activity

In Tables III.5 and III.6 the two classifications have been kept apart, the population being classified first by age and second by activity. These two classifications can be allowed to interact. If they are, and if the classification by age is regarded as the primary classification, the result can be set out as in Table III.7. In this table the secondary classification is applied only to the opening and closing stocks, with the convention that where no subdivision is made, none of the survivors in question is at school.

In Table III.7 the distinction between the two activities first appears in the outflow at age 2: no children were at school at the beginning of the year in which they had their second birthday, but 1,300 of them had

TABLE III.7. DEMOGRAPHIC ACCOUNTS FOR ACTUAL RESIDENTS, CLASSIFIED BY AGE AND ACTIVITY

ENGLAND AND WALES, MALE POPULATION, 1965

*Thousands.*

| Inflows | | | Outflows | | |
|---|---|---|---|---|---|
| **AGE 0** | | | | | |
| Births in 1965 | | 443.2 | Deaths in 1965 | | 8.3 |
| Net immigrants in 1965 | | 0.0 | Survivors to 1966 | | 434.9 |
| Total | | 443.2 | Total | | 443.2 |
| **AGE 1** | | | | | |
| Survivors from 1964 | | 442.2 | Deaths in 1965 | | 1.5 |
| Net immigrants in 1965 | | 0.0 | Survivors to 1966 | | 440.7 |
| Total | | 442.2 | Total | | 442.2 |
| **AGE 2** | | | | | |
| Survivors from 1964 | | 428.0 | Deaths in 1965 | | 0.5 |
| | | | Survivors to 1966: | | |
| Net immigrants in 1965 | | 0.5 | at school | 1.3 | |
| | | | not at school | 426.7 | |
| | | | | | 428.0 |
| Total | | 428.5 | Total | | 428.5 |
| **AGE 3** | | | | | |
| Survivors from 1964: | | | Deaths in 1965 | | 0.3 |
| at school | 1.5 | | Survivors to 1966: | | |
| not at school | 418.5 | | at school | 13.0 | |
| | | 420.0 | not at school | 407.0 | |
| Net immigrants in 1965 | | 0.3 | | | 420.0 |
| Total | | 420.3 | Total | | 420.3 |
| **AGE 4** | | | | | |
| Survivors from 1964: | | | Deaths in 1965 | | 0.3 |
| at school | 12.8 | | Survivors to 1966: | | |
| not at school | 394.2 | | at school | 114.2 | |
| | | 407.0 | not at school | 291.8 | |
| Net immigrants in 1965 | | −0.7 | | | 406.0 |
| Total | | 406.3 | Total | | 406.3 |
| **AGES 5-15** | | | | | |
| Survivors from 1964: | | | Deaths in 1965 | | 1.8 |
| at school | 3,561.6 | | Survivors to 1966: | | |
| not at school | 302.3 | | at school | 3,714.5 | |
| | | 3,863.9 | not at school | 143.7 | |
| Net immigrants in 1965 | | −3.9 | | | 3,858.2 |
| Total | | 3,860.0 | Total | | 3,860.0 |
| **AGES 16-19** | | | | | |
| Survivors from 1964: | | | Deaths in 1965 | | 1.6 |
| at school | 417.7 | | Survivors to 1966: | | |
| not at school | 1,146.6 | | at school | 196.9 | |
| | | 1,564.3 | not at school | 1,363.7 | |
| Net immigrants in 1965 | | −2.1 | | | 1,560.6 |
| Total | | 1,562.2 | Total | | 1,562.2 |
| **AGES 20+** | | | | | |
| Survivors from 1964: | | | Deaths in 1965 | | 268.0 |
| at school | 2.1 | | | | |
| not at school | 16,013.1 | | Survivors to 1966 | | 15,769.1 |
| | | 16,015.2 | | | |
| Net immigrants in 1965 | | 21.9 | | | |
| Total | | 16,037.1 | Total | | 16,037.1 |

started school by the end of the year. At age 3, we find 1,500 children at school at the beginning of the year and 13,000 at the end; and at age 4, the opening figure is 12,800 and the closing figure is 114,200.

In the age-group 5 to 15 most children both enter and leave the year at school. Of the 302,300 children who were not at school at the beginning of the year, the great majority was composed of those who had their fifth birthday, and thus reached the age of compulsory education, in the course of the year. Of the 143,700 children who were not at school at the end of the year, the majority were 15-year-olds who left school as soon as they had reached the age at which school attendance is no longer compulsory. In the remaining two age-groups the numbers at school diminish rapidly, until at age 20+ there are none left.

So much for a simple accounting presentation. If we wish to set out the system in greater detail there is everything to be said for using matrices. Let us first consider matrices based on opening and closing stocks and then matrices based on year-to-year flows.

## 7. Matrices Based on Opening and Closing Stocks

These matrices consist of two interconnected but distinct sets of rows and columns, one set relating to our country, the other to the outside world. The rows for our country contain the opening stock of population, the columns contain the closing stock. The rows for the outside world contain, in principle, the births and immigrations of the period, and the columns contain, in principle, the deaths and emigrations of the period. In practice, since it is impossible at present to give a detailed account of migrants, the outside world has only one row, containing births and net immigrations, and one column, containing deaths. A matrix of this kind in its most condensed form, with no distinctions of age or activity, is given in Table III.8.

Table III.8. A CLOSED (SINGLE-YEAR) DEMOGRAPHIC MATRIX FOR ACTUAL RESIDENTS
England and Wales, male population, 1965

*Thousands.*

| | Outside world | England and Wales | Total |
|---|---|---|---|
| Outside world | 8.3 | 450.9 | 459.2 |
| England and Wales | 274.0 | 22,866.6 | 23,140.6 |
| Total | 282.3 | 23,317.5 | 23,599.8 |

In Table III.8 the row and column totals correspond respectively to the left and right-hand sides of Tables III.4, except that here births and net immigrations have been added together: 443.2 + 16.0 = 459.2.

The first row shows that of this total inflow of 459,200 from the outside world, 8,300 died within the year and 450,900 survived to the end of the year; those who died are shown as flowing from the outside world directly back to the outside world because, although they were present in

the country for a little while, they never appeared in any year-end count of the population.

The second row shows that of the total inflow of 23,140,600 survivors from the preceding year, who formed the opening stock of 1965, 274,000 died during the year and 22,866,600 survived to the end of the year.

The first column shows that of the total outflow of 282,300 deaths to the outside world, 8,300 had come in during the year from the outside world and 274,000 were survivors from the preceding year.

The second column shows that of the total outflow of 23,317,500 survivors into the following year, who formed the closing stock of 1965, 450,900 had come in from the outside world during the year and 22,866,600 were survivors from the preceding year.

The final figure of 23,599,800, in the bottom right-hand corner of the table, represents the total flows into and out of 1965.

One could of course set up matrices corresponding to Tables III.5 and III.6, but this seems hardly necessary. Instead, we shall jump straight to the matrix analogue of Table III.7. This is given in Table III.9.

In Table III.9 the rows and columns of Table III.8 have been subdivided first by age and then, within each age, by activity. In this table we can trace changes of activity taking place within the year. For example, if we look at the second row for age 4 we can see that of the opening stock of 394,200 boys who were not at school at the beginning of 1965, 300 died in the course of the year and the remainder survived to the end of the year; of these, 101,400 entered school during the year and 292,500 stayed at home. Similarly, of the opening stock of 3,561,600 boys in the age-group 5-15 who were at school at the beginning of the year, 1,400 died during the year, 3,438,100 continued at school and 122,100 left school.

A set of transition coefficients can be calculated from Table III.9 by dividing the elements in each row by the row total. These coefficients show the proportions in which the opening stock of people of a particular age found in a particular activity at the beginning of the year either die during the year or redistribute themselves among the various activities open to them. For example, of the 394,200 boys in the 4-year-old age bracket who were not at school at the beginning of 1965, $0.3/394.2 = 0.0008$ died within the year; $101.4/394.2 = 0.2572$ entered school during the year; and $292.5/394.2 = 0.7420$ stayed at home.

## 8. Matrices Based on Year-to-Year Flows

These matrices too consist of two sets of rows and columns, one for our country and one for the outside world, but the sets are placed in a different time-relationship to each other. The rows for our country show the outflows of last year, composed partly of the deaths of last year and partly of the survivors into this year; and the columns for our country show the inflows into this year, composed partly of last year's survivors and partly of this year's births and net immigrations. The deaths of last year appear in the column for the outside world, and the births and net immigrations of this year appear in the row for the outside world. In other words, a time-lag is introduced which links explicitly two successive periods. One such matrix is sufficient to calculate the transition proportions of one

TABLE III.9. A CLOSED (SINGLE-YEAR) DEMOGRAPHIC MATRIX FO
ENGLAND AND WALE

| | | | Outside world | Englar | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Age 0 | Age 1 | Age 2 | | Age 3 | |
| | | | | Not at school | | At school | Not at school | At school | Not at school |
| Outside world | | | 8.3 | 434.9 | 0.0 | | 0.5 | | 0. |
| England and Wales | Age 1 | Not at school | 1.5 | | 440.7 | | | | |
| | Age 2 | At school<br>Not at school | 0.5 | | | 1.3 | 426.2 | | |
| | Age 3 | At school<br>Not at school | 0.3 | | | | | 1.5<br>11.5 | 406. |
| | Age 4 | At school<br>Not at school | 0.3 | | | | | | |
| | Ages 5-15 | At school<br>Not at school | 1.4<br>0.4 | | | | | | |
| | Ages 16-19 | At school<br>Not at school | 1.6 | | | | | | |
| | Ages 20 + | At school<br>Not at school | 268.0 | | | | | | |
| Total | | | 282.3 | 434.9 | 440.7 | 1.3 | 426.7 | 13.0 | 407. |

year, say 1964-65. But in order to exhibit fully the accounts for 1965 it is necessary, in this method of presentation, to set up a matrix with rows for 1964 and 1965 and columns for 1965 and 1966. Table III.10 gives the analogue of Table III.8 drawn up on these principles.

Table III.10 shows that the total outflow of 1964 was 23,415,400; of this total, 274,800 flowed to the outside world, that is died in 1964, and 23,140,600 flowed into the succeding period, that is survived to form the opening stock of 1965. The inflow into 1965, totalling 23,599,800, consisted of the 23,140,600 survivors from 1964 plus the births and immigrations of 1965, amounting to 459,200. Of this total inflow, 282,300 died in 1965 and 23,317,500 survived to form the opening stock of 1966; in which year they were joined by births and net immigrants amounting to 450,000, making a total inflow into 1966 of 23,767,500.

On comparing this table with Table III.8 it will be seen that the column for 1965, showing the total inflows into the year, corresponds to the final column of Table III.8; and that the row for 1965, showing the total outflows of the year, corresponds to the final row of Table III.8.

CTUAL RESIDENTS CLASSIFIED BY AGE AND ACTIVITY
ALE POPULATION, 1965

*Thousands.*

and Wales

| Age 4 | | Ages 5-15 | | Ages 16-19 | | Ages 20 + | | Total |
|---|---|---|---|---|---|---|---|---|
| At school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | |
| | −0.7 | −5.3 | 1.4 | · | −2.1 | | 21.9 | 459.2 |
| | | | | | | | | 442.2 |
| | | | | | | | | 0<br>428.0 |
| | | | | | | | | 1.5<br>418.5 |
| 12.8<br>101.4 | 292.5 | | | | | | | 12.8<br>394.2 |
| | | 3,438.1<br>281.7 | 122.1<br>20.2 | | | | | 3,561.6<br>302.3 |
| | | | | 196.9 | 220.8<br>1,145.0 | | | 417.7<br>1,146.6 |
| | | | | | | | 2.1<br>15,745.1 | 2.1<br>16,013.1 |
| 114.2 | 291.8 | 3,714.5 | 143.7 | 196.9 | 1,363.7 | 0 | 15,769.1 | 23,599.8 |

TABLE III.10. AN OPEN (YEAR-TO-YEAR) DEMOGRAPHIC MATRIX FOR ACTUAL RESIDENTS
ENGLAND AND WALES, MALE POPULATION, 1964-65 AND 1965-66

*Thousands.*

| | | Outside world | England and Wales | | Total outflows |
|---|---|---|---|---|---|
| | | | 1965 | 1966 | |
| Outside world ..... | | | 459.2 | 450.0 | |
| England and Wales | 1964 ..... | 274.8 | 23,140.6 | | 23,415.4 |
| | 1965 ..... | 282.3 | | 23,317.5 | 23,599.8 |
| Total inflows ..... | | | 23,599.8 | 23,767.5 | |

TABLE III.11. AN OPEN (YEAR TO-YEAR) DEMOGRAPHIC MATRIX FOR

| | | | Outside world | England and Wales, 1965 | | | | | | | | | | | | | sc |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Age 0 | Age 1 | Age 2 | | Age 3 | | Age 4 | | Ages 5-15 | | Ages 16-19 | | | |
| | | | | Not at school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | | |
| Outside world | | | | 443.2 | | | 0.5 | | 0.3 | | −0.7 | −5.3 | 1.4 | | −2.1 | | |
| England and Wales 1964 | Age 0 | Not at school | 8.9 | | 442.2 | | | | | | | | | | | | |
| | Age 1 | Not at school | 1.4 | | | 1.3 | 426.7 | | | | | | | | | | |
| | Age 2 | At school | | | | | | 1.5 | | | | | | | | | |
| | | Not at school | 0.5 | | | | | 11.5 | 407.0 | | | | | | | | |
| | Age 3 | At school | | | | | | | | 12.8 | | | | | | | |
| | | Not at school | 0.3 | | | | | | | 101.4 | 292.8 | | | | | | |
| | Age 4 | At school | | | | | | | | | | 112.5 | | | | | |
| | | Not at school | 0.2 | | | | | | | | | 273.0 | 8.5 | | | | |
| | Ages 5-15 | At school | 1.5 | | | | | | | | | 3,327.0 | 122.1 | 106.1 | 116.2 | | |
| | | Not at school | 0.2 | | | | | | | | | 8.9 | 11.9 | | 139.7 | | |
| | Ages 16-19 | At school | | | | | | | | | | | | 90.8 | 104.6 | | |
| | | Not at school | 1.5 | | | | | | | | | | | | 1,006.9 | | |
| | Ages 20 + | At school | | | | | | | | | | | | | | | |
| | | Not at school | 260.3 | | | | | | | | | | | | | | |
| England and Wales 1965 | Age 0 | Not at school | 8.3 | | | | | | | | | | | | | | |
| | Age 1 | Not at school | 1.5 | | | | | | | | | | | | | | |
| | Age 2 | At school | | | | | | | | | | | | | | | |
| | | Not at school | 0.5 | | | | | | | | | | | | | | |
| | Age 3 | At school | | | | | | | | | | | | | | | |
| | | Not at school | 0.3 | | | | | | | | | | | | | | |
| | Age 4 | At school | | | | | | | | | | | | | | | |
| | | Not at school | 0.3 | | | | | | | | | | | | | | |
| | Ages 5-15 | At school | 1.6 | | | | | | | | | | | | | | |
| | | Not at school | 0.2 | | | | | | | | | | | | | | |
| | Ages 16-19 | At school | | | | | | | | | | | | | | | |
| | | Not at school | 1.6 | | | | | | | | | | | | | | |
| | Ages 20 + | At school | | | | | | | | | | | | | | | |
| | | Not at school | 268.0 | | | | | | | | | | | | | | |
| Total | | | | 443.2 | 442.2 | 1.3 | 427.2 | 13.0 | 407.3 | 114.2 | 292.1 | 3,716.1 | 143.9 | 196.9 | 1,365.3 | | |



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X FOR ACTUAL RESIDENTS CLASSIFIED BY AGE AND ACTIVITY

*Thousands.*

| | England and Wales, 1966 | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Ages 20 + | | Age 0 | Age 1 | Age 2 | | Age 3 | | Age 4 | | Ages 5-15 | | Ages 16-19 | | Ages 20 + | | Total |
| at ol | At school | Not at school | Not at school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | At school | Not at school | At school | Not school | |
| 2.1 | | −2.9 | 437.3 | 0.0 | | −0.2 | | −0.7 | | −0.7 | −0.4 | | | 1.6 | | 13.1 | |
| | | | | | | | | | | | | | | | | | 451.1 |
| | | | | | | | | | | | | | | | | | 429.4 |
| | | | | | | | | | | | | | | | | | 1.5<br>419.0 |
| | | | | | | | | | | | | | | | | | 12.8<br>394.5 |
| | | | | | | | | | | | | | | | | | 112.5<br>281.7 |
| 6.2<br>.7 | | | | | | | | | | | | | | | | | 3,672.9<br>160.7 |
| .6<br>.9 | | 2.1<br>331.9 | | | | | | | | | | | | | | | 197.5<br>1,340.3 |
| | | 15,681.2 | | | | | | | | | | | | | | | 0.0<br>15,941.5 |
| | | | | 434.9 | | | | | | | | | | | | | 443.2 |
| | | | | | 1.3 | 439.4 | | | | | | | | | | | 442.2 |
| | | | | | | | 1.3<br>14.1 | 412.6 | | | | | | | | | 1.3<br>427.2 |
| | | | | | | | | | 13.0<br>111.3 | 295.7 | | | | | | | 13.0<br>407.3 |
| | | | | | | | | | | | 114.2<br>282.1 | 9.7 | | | | | 114.2<br>292.1 |
| | | | | | | | | | | | 3,382.2<br>6.4 | 112.6<br>11.3 | 107.0 | 112.7<br>126.0 | | | 3,716.1<br>143.9 |
| | | | | | | | | | | | | | 91.5 | 102.5<br>977.3 | | 2.9<br>386.4 | 196.9<br>1,365.3 |
| | | | | | | | | | | | | | | | | 15,769.1 | 0.0<br>16,037.1 |
| 5.3 | 0.0 | 16,037.1 | 437.3 | 434.9 | 1.3 | 439.2 | 15.4 | 411.9 | 124.3 | 295.0 | 3,784.5 | 133.6 | 198.5 | 1,320.1 | 0.0 | 16,171.5 | |

If the figures in Table III.10 are subdivided by age and activity, the result is Table III.11, which is the analogue of Table III.9.

In Table III.11, the age of the outflows is defined by the date specified against the rows, and the age of the inflows by the date specified above the columns; thus in the upper left-hand quarter of the table, the age in the rows is "age at the end of 1964"; and the age in the columns is "age at the end of 1965". In this table we can trace year-to-year changes of state. That is to say, we can see how the people who flow out of a particular combination of age and activity in a given year either die in that year or flow into different combinations of age and activity in the following year. The headings of the table should provide a sufficient guide to the dynamics of the system; but in case they do not, the following two paragraphs may prove useful.

Let us start with an example of flows into and out of single ages. If we look at the row for age 1 in 1964, we can see that of the 429,400 boys who flowed out of that category, 1,400 died in 1964 and 428,000 survived to 1965, when they had their second birthday; of these survivors, 1,300 entered school in the course of 1965 and 426,700 stayed at home. From the column for "age 2 not at school" in 1965 we can see that the number of those who stayed at home was increased during the year by 500 net immigrants of the same age; thus the inflow into the state "age 2 not at school" in 1965 totalled 427,200. If we now look at the row for "age 2 not at school" in 1965 we find that, of this total, 500 died in 1965 and 426,700 survived into 1966, when they had their third birthday; of these survivors, 14,100 entered school in 1966 and 412,600 stayed at home. Finally, from the column for "age 3 not at school" in 1966 we see that the number of those who stayed at home was diminished in the course of that year by −700 net immigrants of the same age, making a total inflow of 411,900.

Let us now look at an example of flows into and of age-groups. Taking as a starting point the outflow of 4-year-olds at school in 1964, we can see that it totalled 112,500 boys and that they all survived into 1965, when they had their fifth birthday and thus entered the age-group 5-15, and that all of them remained at school. Looking next at the column for ages 5-15 at school in 1965, we see that this was only one of several inflows into that state: 273,000 flowed in from 1964 as 4-year-olds not at school; 3,327,000 flowed in as 5-14-year-olds at school; 8,900 flowed in as 5-14-year-olds not at school; and there was a negative inflow of −5,300 net immigrants; thus the total inflow into the 5-15 age-group at school in 1965 was 3,716,100. From the corresponding row for 1965 we see that, of this total, 1,600 died within the year and that the survivors into 1966 divided themselves into four different states: 3,382,200 continued in the same age-group and remained at school; 112,600 continued in the same age-group but left school; 107,000 entered the 16-19 age-group and remained at school; and 112,700 entered the 16-19 age-group and left school. Finally, from the four columns for ages 5-15 and 16-19 in 1966 we see how these four flows combined with others to form the total inflows into the four relevant states in 1966.

So much for the descriptive uses of Table III.11. For analytical purposes, a set of transition proportions can be calculated from it exactly as it could from Table III.9, that is by dividing the elements in each row by the row total. In this case the calculations show the proportions in which

the people flowing out of a particular state in a given year either flow out through death within the year or enter the various states open to them in the following year. For example, of the 3,716,100 boys who were aged 5-15 and at school when they flowed out of 1965, the proportion who died in the year was 1.6/3,716.1 = 0.0004; the proportion who continued in the same age-group and remained at school in 1966 was 3,382.2/3,716.1 = 0.9101; and the proportion who moved into the 16-19 age-group and remained at school was 107.0/3,716.1 = 0.0288.

The differences between the type of matrix illustrated in Table III.9 and that illustrated in Table III.11 are more a matter of approach than of substance. The flow and stock totals in the two tables are exactly the same, but placed in a different relationship to each other, depending on the point at which one has chosen to record deaths: in Table III.9 the deaths of the year (1965) are recorded at the outset, and only those who survive to the end of the year are allowed to change state during the year; in Table III.11, on the other hand, the whole of the opening stock goes through the process of redistribution, and the deaths of the year (1965) are recorded at the next round. In the first case, some people are shown as dying before they have reached their final activity; in the second case, some are shown as surviving in their final activity beyond the time of their death. Which convention one adopts does not much matter: the transition proportions obtainable from a table like Table III.9 would, in any normal circumstances, tell much the same story as those obtainable from the upper half of a table like Table III.11.

A more substantial difference lies in the recording of age. Since the recording of age for any group depends on the date of the period of account, a matrix that refers to a single year can only record one age, or one age-range, for each group. From this it follows that in this kind of matrix only the changes of activity that take place during a year be shown; the changes of age cannot. For example, in Table III.9 the opening stock in the rows for age 4 appears as aged 4 right from the start, whereas in reality when it flowed into the year it was still aged 3. This does not matter when one is dealing with single ages. But when it comes to recording the flows into and out of age-groups, as opposed to single ages, it is an advantage to be able to show the inflows at the younger end of the group and the outflows at the older end; for example, the 4-year-olds of 1964 flowing into the 5 to 15-year-olds of 1965, and the 15-year-olds flowing into 16 to 19-year-olds. In Table III.9 this detail is lost.

## 9. Vintages

The tables given so far all relate to the total population flowing into and out of a particular year. They provide, therefore, cross-section analyses of all the vintages, or cohorts, alive in that year. By selecting the data for one of a sequence of ages from a number of consecutive tables, it is possible to trace through time the maturation of a vintage. Unlike the cross-section tables, no vintage table that does not cover a whole life-span can be said to be complete: the children born in 1960, for example, will in 1966 only have reached the age of 6.

Vintage tables can be set out both in the form of accounts and in the form of matrices, and can be based either on opening and closing

TABLE III.12. A DEMOGRAPHIC MATRIX FOR THE 1960 VINTAGE BASED ON OPENING AND CLOSING STOCKS

ENGLAND AND WALES, MALE POPULATION, 1960-66

*Thousands.*

| | | Outside world | England and Wales | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1960 Age 0 | 1961 Age 1 | 1962 Age 2 | 1963 Age 3 | 1964 Age 4 | 1965 Age 5 | 1966 Age 6 | |
| Outside world ..... | | 8.9 | 395.3 | 0.0 | 1.7 | 0.3 | −0.8 | −0.8 | −0.8 | |
| England and Wales | 1961. Age 1 | 1.5 | | 393.8 | | | | | | 395.3 |
| | 1962. Age 2 | 0.5 | | | 393.3 | | | | | 393.8 |
| | 1963. Age 3 | 0.3 | | | | 394.7 | | | | 395.0 |
| | 1964. Age 4 | 0.2 | | | | | 394.8 | | | 395.0 |
| | 1965. Age 5 | 0.2 | | | | | | 393.8 | | 394.0 |
| | 1966. Age 6 | 0.2 | | | | | | | 392.8 | 393.0 |
| | 1967. Age 7 | 0.2 | | | | | | | | (392.0) |
| Total ..... | | | 395.3 | 393.8 | 395.0 | 395.0 | 394.0 | 393.0 | 392.0 | |

stocks or on year-to-year flows. Two alternative matrices for the 1960 vintage age given in Tables III.12 and III.13.

The difference between the two tables is obvious. In Table III.12 the totals of the row and column for each year represent respectively the opening and closing stocks of the year and therefore do not balance. What does balance are the closing stock of one year and the opening stock of the next: the closing stock of 0-year-olds in the column for 1960, totalling 395,300, is the same as the opening stock of 1-year-olds in the row for 1961.

In Table III.13, on the other hand, the row and column totals for each year represent respectively the total outflows and the total inflows of the year, and therefore they balance. The identity between the closing stock of one year and the opening stock of the next is brought out in the central part of the table: the 395,300 0-year-old survivors of 1960 flow directly into the 1-year-olds of 1961 at the intersection of the 1960 row and the 1961 column.

These two tables are the vintage analogues of Tables III.8 and III.10. If they were subdivided by activity they would become the vintage analogues of Tables III.9 and III.11.

In order to develop an example of vintage accounts to the degree of detail described in later chapters, it would be necessary to follow the maturing of the 1948 vintage, that is to say to have full data for twenty consecutive years. These data are not readily available, and for this reason the subject of vintages will not be pursued any further in this report.



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TABLE III.13. A DEMOGRAPHIC MATRIX FOR THE 1960 VINTAGE BASED ON YEAR-TO-YEAR FLOWS
ENGLAND AND WALES, MALE POPULATION, 1960-66

*Thousands.*

| | | Outside world | England and Wales | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1960 Age 0 | 1961 Age 1 | 1962 Age 2 | 1963 Age 3 | 1964 Age 4 | 1965 Age 5 | 1966 Age 6 | |
| Outside world ..... | | | 404.2 | 0.0 | 1.7 | 0.3 | −0.8 | −0.8 | −0.8 | |
| England and Wales | 1960. Age 0 | 8.9 | | 395.3 | | | | | | 404.2 |
| | 1961. Age 1 | 1.5 | | | 393.8 | | | | | 395.3 |
| | 1962. Age 2 | 0.5 | | | | 395.0 | | | | 395.5 |
| | 1963. Age 3 | 0.3 | | | | | 395.0 | | | 395.3 |
| | 1964. Age 4 | 0.2 | | | | | | 394.0 | | 394.2 |
| | 1965. Age 5 | 0.2 | | | | | | | 393.0 | 393.2 |
| | 1966. Age 6 | 0.2 | | | | | | | | (392.2) |
| Total ..... | | | 404.2 | 395.3 | 395.5 | 395.3 | 394.2 | 393.2 | 392.2 | |

## 10. FURTHER DEVELOPMENTS

As we shall see in the chapters that follow, the accounting scheme developed so far could be elaborated in many ways. The recording of individual ages could be continued much further, or even throughout life; the population could be divided not only by sex but also by social class, intelligence, medical history and other family and personal characteristics; the dichotomy between those at school and those not at school could be replaced by a detailed classification of activities; the main field of study in the higher learning activities could be separated out; educational qualifications could be introduced; and, on leaving the educational system, individuals could be classified by industry and occupation.

As in the economic case, it may be very difficult in practice to establish a completely integrated system of demographic accounts. A fully integrated system is useful, however, even if it can be realized statistically only in part, because it shows clearly how concepts are related. In the case of the national economic accounts it is usual to supplement the accounting system by a set of supporting tables drawn from the system itself and thus related to each other. It is also usual to provide analytical tools such as classification converters, input-output coefficient matrices, inverse matrices and so on.

A similar course can be followed in demographic accounting. Tables X.1, X.2 and X.3 below[1] carry the numerical example of this chapter a

1. Under cover flap.

modest step further. The twenty ages from 0 through 19 are shown separately and the number of activities, all but one of which are learning activities is increased to fourteen. Ages and activities interact to define one hundred and fourteen states. Table X.1 sets out the flows between these states from 1964 to 1965 and is simply an elaboration of part of the top left-hand quadrant of Table III.11 above. Table X.2 is a coefficient matrix of transition proportions derived from Table X.1. And Table X.3 is a matrix multiplier based on Table X.2. The nature of these derived tables is described in sections 2 and 3 of Chapter X, and examples of their use are provided in Tables X.4 through X.6.

# IV

## THE STATISTICAL BACKGROUND

### 1. Types of Information

The information available for constructing demographic accounts varies from country to country and ranges, over different parts of the system, from a limited amount of highly aggregated data on stocks to individualized data which enable the position and movements of every individual to be traced. Since what can be got out of a system of demographic accounts depends on what is put into it, it is useful to consider what can be done with different kinds of information.

By itself, information on stocks is usually insufficient because, while it makes it possible to estimate net inward and outward flows in the aggregate, it does not make it possible to estimate net flows, still less gross flows, between single states. In very simple cases, net flows between states can be inferred but, in general, some explicit information on flows is necessary. With good stock and flow data one can go a long way in the construction of the system; but as the accounts come to be further and further elaborated, the moment is bound to come when individualized data are found to provide the only practicable means of collating the basic records.

The merits and demerits of each type of information are examined in the following three sections.

### 2. Stock Statistics

Stock statistics, even if classified by year of age and by activity and even if accompanied by a certain amount of purely demographic information on flows (births, deaths and migrations), are of only limited value. By comparing the stock aged $\theta$ in a particular activity at the end of a year with the stock aged $\theta$-1 in the same activity at the beginning of the year, it is possible to work out the net change of $\theta$-year-olds in the given activity during the year. At best, if the number of admissible flows is sufficiently limited, it may be possible to infer the magnitude of these flows by using the constraints of the system.

For example, if we turn back to Table III.11 above we can see that of the children who had their third birthday in 1965, 1,500 were at school at the beginning of the year and 13,000 were at school at the end of it. On the assumption that all the children who were at school at the beginning of the year were still at school at the end of it, that is to say that the flow

from "at school" to "not at school" was zero, we can infer that 11,500 of those children went to school for the first time during that year. Further, since the stock of rising 3-year-olds at the beginning of 1965 was 420,000, and since 1,500 of them were already at school, we can infer that the remaining 418,500 were not at school; and since 11,500 of these went to school for the first time during the year, it follows that the remaining 407,000 did not go to school at all.

This argument implies the further assumption that none of the children who went to school emigrated during the year and that their numbers were not increased by immigrants. A comparison of population estimates indicates that in 1965, after making allowance for deaths, there was at this age a net immigration of 300. The whole of this number is shown as an inflow into the group of children who were not at school, since this is by far the larger of the two groups; but this is probably an oversimplification.

This example shows the inevitable inaccuracies to which one is driven, even in the most straightforward cases, if one has only stock statistics to go by.

## 3. Flow Statistics

Flow statistics tell us how the numbers in a given category at a point in time rearrange themselves over the same or other categories over a given interval of time. For example, if we had flow statistics for the 3-year-olds of 1965 we might learn that of the 1,500 who were at school at the beginning of the year, 300 returned home for one reason or another in the course of the year and only 1,200 were still at school at the end of the year. We might also learn that of the 418,500 who were not at school at the beginning of the year, 11,700, rather than the 11,500 inferred from stock statistics, went to school for the first time during the year. Since there were 13,000 3-year-olds at school at the end of the year, this information would imply a net inflow of 100 immigrants into the 3-year-old group at school. Evidently, with accurate information of this kind the inferences drawn from stock statistics could be much improved.

As we progress through the ages, the possible movements gradually become more and more numerous and we reach a point where flow statistics become essential. Consider an age at which school attendance is no longer compulsory, 16 let us say. From stock statistics we may learn that, whereas there were 100,000 boys in a particular type of school at the beginning of the year, the number remaining at the end was only 20,000. The net outflow was therefore 80,000, but we have no means of telling how far this was made up of those leaving the school system altogether and how far those transferring to other types of school. School-leaver statistics might show that there were in fact only 75,000 leavers from that particular type of school, and from this it could be inferred that there was a net transfer to other schools of 5,000. But without flow statistics on transfers it would be very difficult to estimate, simply from the constraints of the system, the proportions in which these 5,000 had redistributed themselves among the different types of school open to them. If we happen to have specific knowledge about possible flows, we may be able to estimate approximately the magnitude of the transfers; but even so some uncertainty is bound to remain.

Flow statistics may be of two kinds: they may record the flow out of

a category to all destinations or into it from all sources; or they may record the flow out of one category into another. The first kind gives rise to the problem of matching inflows and outflows: of discovering, for example, the sources of admissions to universities. Without some information of the second kind it is virtually impossible to resolve this problem, since even if the system contains information on educational qualifications, it is well known that not all students who qualify for entry into a university actually apply for, or obtain, a place. In the absence of some such information, there is little to be done but make use of the device of a dummy account which collects together the whole of a certain type of outflow, say school leavers of a particular age, and reallocates them as a group over their various destinations. The same applies to the flows into employment: without a better integration of educational and manpower statistics than usually exists, it is necessary to resort to this device in order to link the two.

Flow statistics which link one activity to another may provide either backward links or forward links. Backward links record the position last year of someone whose position this year is known. Forward links, on the other hand, record the expected position next year of someone whose position this year is known. For example, in England the school-leaver statistics classify leavers according to the activities they are expected to enter on leaving, as these are generally known to the schools which provide the information. Such information is much better than nothing, but even so one cannot absolutely rely on it: while, on the whole, a school leaver who has gained a place in a university can be expected to take it up, a leaver who has expressed the intention of becoming an apprentice in the building trade, say, may be unable to do so or may find a better opening elsewhere or may simply change his mind.

Information on flows may be collected either from institutions or from individuals. Institutions are likely to possess only a limited amount of information about individuals, the information that they consider necessary for their own operations; in general, they will not find it possible to record complete educational histories. Moreover, if institutions are used as collecting units, it is necessary to obtain returns from every type of institution, if only on a sample basis, and this may give rise to serious practical problems. These remarks are not intended as arguments against the collection of information from institutions but only as a reminder that this form of enquiry is more likely to be practicable if it concentrates on year-to-year links without much reference to the past.

The use of individuals as the collecting unit would, in principle, overcome these difficulties since most individuals could reconstitute their educational history. But in practice the difficulties would probably be considerable and the cost excessive. If the system is to have a memory, for example is to be able to distribute this year's entrants into universities according to their educational position three years ago, then a somewhat different organisation of data collection would seem to be desirable.

## 4. Registration Numbers and Individualized Data Systems

The essence of the problem just outlined is to be able to keep together information from separate intervals of time: to be able to record that an individual who passed from state $r$ to state $s$ in a given interval

had passed from $q$ to $r$ in the preceding interval and from $p$ to $q$ in the interval before that. This is equivalent to being able to compile a record of the position in which the individual is to be found at the beginning of each interval throughout his life.

It is not necessary that all the information should be collected from any one source, whether it be the individual himself, his family or one of the numerous institutions to which he is attached in the course of his life, provided that the individual is given a means of identification such as a registration number. If this number is used in all records relating to the individual, it is possible, with the help of computers, to merge these records so as to reconstitute complete individual histories. Since most of the information desired is likely in fact to be recorded somewhere, this arrangement can be regarded simply as a system of data retrieval and integration.

In limited fields, such as university attendance, where the number of states to be distinguished is large, such individualized data systems are likely to prove of the greatest value. However, their extension to cover many aspects of economic and social life runs into difficulties. These arise not so much from the purely mechanical difficulties of handling enormous quantities of data, though these are real enough, as from the political dangers that accompany the possibility of assembling complete individual dossiers. These dangers are inherent in any co-ordinated information system, such as questions relating to name, address and race on a census form, but they would certainly be increased by an indiscriminate widening of the possibility to merge records. This conflict between the advantages and the dangers of detailed individual records poses a serious problem.

# V

# THE ACTIVE SEQUENCE: LEARNING ACTIVITIES

## 1. The Starting Point

From a purely formal point of view, introducing learning activities into the demographic accounts is a simple matter: given $n$ learning activities to be distinguished, the relevant ages are subdivided into $n+1$ categories, the supplementary category being required to accommodate the people in each age who are not educationally active. The total flows into and out of each age can then be deconsolidated so as to show the flows into, through and out of each activity.

From an operational point of view, however, this apparently simple exercise comes up against a number of difficulties, both conceptual and practical. This chapter is mainly concerned with the former: how should learning activities be defined; which activities should be accounted for and which should not; and how should those that are accounted for be classified. Practical problems are touched upon only in so far as they impinge on the solution of the conceptual ones.

In this discussion no attempt is made to go into too many taxonomic niceties. At this stage of the development of the subject it seems best to concentrate on the main distinctions to be made, without insisting on the immediate reconciliation of detailed classifications and nomenclature. This should be easy to achieve once the basic principles have been agreed on.

## 2. The Educational Boundary

The definition of learning activities raises the same kind of problem as does the definition of productive activities in the economic accounts. In the economic case it is helpful to think of a boundary, the production boundary, drawn round those activities which constitute production as measured in the economic accounts. In a similar way, it is helpful to think of an educational boundary drawn round those activities which we consider should be recorded under the heading of education.

The core of the concept of education is systematic instruction leading to the acquisition of a specific body of knowledge or a specific skill. This, presumably, is what is generally meant by formal education. Formal education is usually linked with some kind of formal qualification: a school-leaving certificate, a diploma in domestic science, a university degree. Whether the individuals who follow the courses actually take the final

tests and obtain the appropriate qualifications is irrelevant, however: anybody who follows a regular course is receiving formal education and therefore has a place inside the educational boundary. By contrast, the acquisition of knowledge by irregular and sporadic means should be placed outside the boundary.

So much for the theory. Now for the practice.

By far the greater part of formal education takes place in institutions established for the purpose: schools, technical colleges, universities and so on. Schools are of various kinds and some kinds are much better documented than others. In most countries, the great majority are maintained by the central government or by local authorities, and these are usually fairly well documented. In addition there is often a group of private schools which are partly subsidized by the state and which accept some degree of state inspection; these are often educationally important and have an interest in providing information. Beyond these there is likely to be a group of unrecognized schools, of varying educational importance, about which relatively little is known. All such establishments, which provide full-time schooling, should be placed inside the educational boundary, however difficult it may be to do so.

Establishments of further education are even more differentiated and pose even greater practical problems. Again, there are the straight-forward cases, like the great universities. But there is also a wide range of institutions which are not characterized by a common purpose or mode of operation: some are specialized in a set of narrow vocational disciplines and raise the question of educational content; others operate in terms of very short courses, which complicates the recording of inflows and outflows in annual accounts. Still, all establishments of further education, including this difficult fringe, belong inside the boundary.

A complement to formal education which should not be ignored is private tuition by qualified teachers. Not only is it a common remedy for bad performance at school, it is also a widespread means of learning subjects which at school are either not taught at all or cannot be fitted into the time-table of a particular curriculum. Two obvious examples are private language lessons and private music lessons; indeed, the latter are the foundation of many musical careers. Although recording private education, even approximately, may be impossible with the information available at present, there is little doubt that its contribution to formal education is quite substantial, especially at the primary and secondary levels.

Much further education of a professional nature does not take place in establishments of formal education either. A professional association, say of engineers or accountants, may provide a curriculum and a system of regular examinations but little or none of the necessary teaching; while preparing for such examinations, the students will usually be earning their living and will depend largely on spare-time reading, evening classes and correspondence courses to gain the necessary knowledge. This complex of activities is also extremely hard to bring into any regular statistical reporting system, yet in many countries it is the means by which most members of some important professions obtain their qualifications.

So far we have considered activities that *prima facie* lie inside the educational boundary even if some of them are very difficult to measure. Let us now turn to undoubtedly educational activities that lie *prima facie* outside the boundary. In the first place there is the analogue of house-



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hold and amateur activities in the economic case: the parent who teaches the children to read and write before they go to school and helps them with their homework when they are at school. James Mill's considerable educational achievement of teaching Greek to his three-year-old son John Stuart Mill would go unrecorded under this convention.

In the second place, quite a lot is done in the field of technical and professional education by institutions which are not normally associated with education at all. Apart from training on the job, which may contribute more than anything else to many people's knowledge of particular skills, cases can frequently be found where firms or government departments organise specialist courses either for their own members or for a wider public. Thus a gas company may set out at a particular time to teach its employees about the technicalities of converting equipment from town gas to natural gas; a government department may provide refresher courses in economics for administrative civil servants; a central statistical office may run seminars on national income statistics for the benefit alike of official statisticians and of business economists; a firm of consultants may organise conferences on systems analysis and operational research with the aim of widening the understanding of new techniques whose applications they are trying to sell.

We can conclude from this brief survey of the subject that defining educational activities is a complicated matter which illustrates, sometimes in an extreme form, the difficulties of defining productive activity in general. In setting up a recording system, there can be little doubt that a start should be made by drawing the educational boundary fairly tight so as to include only institutions of formal education. At the same time, it is desirable to keep in mind the less regular means by which knowledge can be acquired and try to work them gradually into the system, at least at the theoretical level. Although the wish to record these less regular means at the present time may be little more than the expression of a pious hope, they are certainly important in some fields and may well become more important in the future, especially in view of the increasing rate of obsolescence of technical knowledge and the consequent return to generalism in early education.

### 3. Full-Time Formal Education

If the above argument is accepted, we can go on to divide education into three broad groups: full-time formal education; part-time formal education; and informal education. All three groups contain a proportion of people who are privately taught. In the second and, even more, in the third group this presents a serious recording problem. In the first it does not: the number of children of school age who are fully educated at home, either for reasons of health or because they live too far from a school or simply because their parents prefer it so, is so small that it can be ignored at this stage.

Thus full-time formal education can be identified with the activities that take place in educational institutions. And since institutions are likely to be in most countries the most accessible source of information, it might be advisable at this early stage to adopt institutional categories as our principal classification. Within this institutional classification many subdivisions, reflecting different aspects of the educational process, are

possible: by subject studied, by grade, by qualification obtained, even by curriculum, teaching method and size of class. These taxonomic questions and the practical obstacles they encounter are reviewed below.

a) *Institutions*

A short list of institutions of full-time formal education might run as follows:

nursery and primary schools;
secondary schools;
special schools for the handicapped;
teacher training colleges;
universities and colleges of advanced technology;
other institutions of further education.

The second and last of these categories in particular include a wide range of establishments which vary in nature and nomenclature from one country to another. The aim of what follows is not to review this variety and propose a standard classification but simply to examine the criteria on which such a standard might be based.

One of the main purposes of tracing the career of students into, through and out of the educational system is to determine the options open to a student at each stage of his career and to measure the proportions in which these options are taken up by any particular group of students. For example, we might find that the options open to 16-year-old boys attending a certain type of school were:

*i*) to remain at school;
*ii*) to enter an institution of further education;
*iii*) to take a job.

And we might discover from our demographic accounts that the proportions in which this particular group took up these options were, let us say, 0.81, 0.02 and 0.17 respectively.

These proportions depend not only on age and sex but also on region and, in a marked way, on the type of school attended. This suggests a classification of schools by educational type: distinguishing those that provide various kinds of vocational training from those that provide a general education; and within the latter category, those with modest academic aims from those with high academic aims.

These distinctions do not apply to nursery and primary schools, which offer in all countries a pretty uniform basket of goods, but they do apply to secondary education. In England, for example, one might distinguish four categories of secondary school: "modern", "grammar", "comprehensive" and "other normal". The distinction between modest and high aims is largely reflected in the distinction between secondary modern schools, mainly attended by children who leave school as soon as the law allows them to, and grammar schools, mainly attended by children who intend to pursue their education beyond school-leaving age. Comprehensive schools, a recent creation, are a mixture of the first two types and are rapidly growing in number. The category "other normal schools", as is often the case with "other" categories, has to be introduced in order to prevent the first three from being contaminated by schools which either have very unusual features or are very badly documented. Such rag-bag categories are of course to be avoided as far as possible; but where a group

of schools is badly documented, it may be better to treat it separately and, of necessity, approximately rather than try to combine it with another category.

A type of school somewhat outside this classification are the special schools for handicapped children. This category should be kept separate, since it may be quite large and its transition proportions are likely to be very different from those of other schools.

In the case of institutions of further education, it will be easy to classify those with clear-cut aims and operating at a uniform level, such as teacher training colleges, academies of music and art, military schools, colleges of advanced technology and universities. But it will not be easy to classify the numerous establishments with multiple aims. This group is likely to be concerned with intermediate forms of technical education and may in many ways overlap with the activities of other types of educational establishment. In England, for example, the so-called technical colleges (as opposed to colleges of advanced technology) may combine work at three different levels:

*i*) courses leading to a wide range of technical certificates and diplomas;
*ii*) courses leading to university entrance, corresponding to those held in the senior forms of secondary schools;
*iii*) courses leading to an "external" university degree.

The best way of dealing with such establishments is, wherever possible, to subdivide their activities by type of course; failing this, they had better remain in the category "other".

b) *Subject streams*

Subject streams stand to educational institutions in much the same relationship as commodity groups stand to industries in the economic accounts. Just as commodity groups are defined as the principal products of industries, so can subject streams be defined as the principal products of educational institutions. There is not necessarily a one-to-one correspondence between subject streams and institutions, any more than in economics there is a one-to-one correspondence between commodities and industries; many industries have more than one principal product and many make a number of subsidiary products which are characteristic of other industries.

Similarly, some educational institutions go in for a single subject or complex of subjects: musical academies provide nothing but a musical training, primary schools provide nothing but primary education. Others, while set up to give one type of education to the majority of their pupils, can modify it to meet the requirements of a minority; in England, for example, secondary modern schools provide a grammar-school stream for their best pupils. Such schools could be said to have one principal product, the secondary-modern stream, and one subsidiary one, the grammar-school stream, and the people attending them could be classified accordingly.

From the secondary level up, many institutions offer a number of specialized parallel streams and could thus be said to have several principal products. In English grammar-school education, for example, it becomes desirable around the age of 15 to distinguish at least two parallel streams: the scientific stream, consisting of mathematics and the physical and

biological sciences; and the "arts" stream, which has nothing to do with art but denotes a combination of literary and historical subjects. The rapidly growing group of social sciences, which are comparatively new as school subjects, does not fit readily into this dichotomy; in England it is usually included among the "arts", but as it develops it may come to rate a stream of its own. Also, since a certain number of pupils like to combine scientific and "arts" subjects, it is desirable to have a mixed stream, which might be properly called the humanistic stream. The categories suggested above are appropriate to a type of education with a strong academic bias, but analogous subdivisions can be devised for less academic types of education: artistic (in the real sense of the word), technological, skilled technical, skilled manual, clerical and so on. As people get older, specialization tends to become narrower still and at this stage it is desirable to subdivide the streams into component subjects.

To sum up. At the present stage of the development of educational statistics, institutional categories may constitute the most practical basis for classification from a data-collecting point of view and, if carefully defined, may also give some information on the quality of education. But it seems obvious that from a purely educational point of view a better classification would be by subject stream. This classification would also have considerable practical advantages: it would offer ready pigeonholes for private education when private education became statistically manageable; and, above all, it would make international comparisons much more significant. For these reasons it should perhaps be the principal classification to aim at in the future, when we are nearer to solving the data-collecting problems.

c) *Grades*

Most school systems, the English system is somewhat exceptional in this respect, make use of a sequence of uniform grades: a pupil enters the school system in the first grade and works his way up, taking at each stage some kind of test, success in which is the passport to a higher grade. In the numerical examples given in this paper the activity "at school" is set out in terms of an arrangement whereby ages, but not grades, are recorded, age providing the most convenient link between the various subsystems contained in a general system of demographic accounts. But if we were exclusively interested in the flows of people within the educational subsystem, the activity "at school" could equally well be subdivided by grades; and if there were a one-to-one correspondence between ages and grades the results would be exactly the same as those shown in Tables III.7, III.9 and III.11. Actually, even if all children were to start school at the same age, which they do not necessarily do, it would not be long before differential performance led to a considerable spread of ages in the middle and higher grades. But this would not matter. When it comes to modelling the educational system, virtually the same model, based on transition proportions, can be applied to a grade classification as to an age classification, as can be seen by comparing the analysis given in Chapter IX of this paper with Thonstad's analysis of the educational system of Norway (*Education and Manpower*, Oliver and Boyd, Edinburgh and London, 1969).

Of course it would be possible to record grades as well as ages, grades representing a sub-classification of school types or of subject streams. But it would hardly be useful to introduce both classifications

unless entry into a particular grade were significant for future educational development. The so-called "sixth form" in English schools is a case in point. The sixth form is the nearest thing to a uniform grade in the English school system. It denotes the final stage of secondary education, roughly from the age of 16 to 18. The majority of the pupils who attend it prepare for advanced level ("A-level") examinations, which are a necessary condition for admission to university and to most other institutions of higher education. On the other hand, it is possible to be in the sixth form and not be engaged in this kind of work. For this reason this grade is not introduced in the numerical example given in Chapter X below and in its place a distinction is made at the appropriate ages between the pupils engaged in A-level work and those not so engaged.

Whether one is dealing with ages or grades, it is necessary to consider how far it is desirable to record separately each individual age or grade and how far it is permissible to group them. There are two aspects to this question: how far are we interested in individual ages or grades; and how far is the stability of the transition proportions likely to be affected by grouping. As regards the first aspect, it seems clear that our interest increases with age or grade; there can be relatively little interest in the earlier stages of compulsory schooling, when the only option is usually to remain in the same school either in the same or in a higher grade. As regards the second aspect, the answer must depend on the stability of the birth rate and, to a lesser extent, on the stability of the rates of inward and outward migration. In a stable or exponentially growing population the weights to be applied to age-specific or grade-specific transition proportions would be constant, and therefore grouped transition proportions would be no less stable than those for individual ages or grades. If, however, there are marked fluctuations in the birth or migration rates from year to year, grouped proportions will tend to be less stable than age- or grade-specific ones.

d) *Tests and qualifications*

Apart from a knowledge of subjects studied and progress made, it is also necessary to know something about tests taken and qualifications obtained, particularly at the later stages of the educational process. It is desirable to record at least the final test passed at the end of an educational career, whether this be a leaving certificate from a secondary school or a post-graduate degree from a university.

Of course it is possible to have a system in which people are allowed to leave school without taking any final tests. In such cases, grades might stand for qualifications. In England, for example, the school-leaving age is 15, with no compulsory leaving test. If there were a sequence of uniform grades in the lower range of secondary school, it would at least be possible to record that so many 15-year-olds left while in the eleventh grade, say, so many while in the tenth and so many while still in the ninth.

Alternatively, we may have no information at all about qualifications. This, again, is a situation in which recording grades might be useful. For example, in England in order to enter the sixth form it is necessary to have passed a certain number of ordinary level ("O-level") examinations. Thus if we recorded sixth-form attendance we could see, in the absence of

explicit information about qualifications, how many people had at least a minimum number of O-level passes.

e) *Curricula, teaching methods, size of classes*

Information on these topics is mainly of interest from a pedagogical point of view, when discussing how to adapt an educational system to handle a fast-increasing flow of pupils or how to educate existing numbers more efficiently. It is not very relevant to the problem of how to integrate educational statistics into a general demographic system, and will therefore not be discussed in this paper.

f) *Administrative categories*

The classification envisaged so far relates to the type of education provided and cuts right across administrative categories. From a purely educational point of view it seems the best to adopt. But part of the purpose of studying transition proportions is to estimate the future inputs and costs associated with the changing pattern of education. While these economic and financial factors are probably more effectively studied in terms of an educational classification, they must also be related to an administrative classification since the sources of finance, if not the quantity and composition of the inputs themselves, may vary widely. For instance, English grammar schools may belong to three administrative types: (*i*) maintained by local authorities as part of the state-school system; (*ii*) private non-profit institutions with or without a measure of state assistance; and (*iii*) private profit-making enterprises. Eton, Winchester and all the other great English "public" schools, which were originally founded as charities, belong to the second type.

Any system of statistical compilation should be capable of serving many purposes but is unlikely to be equally suitable for all of them. In the present case there seems little doubt that the principal classification adopted should be on educational lines and that a transforming matrix, or classification converter, should be used to turn educational into administrative categories. Such a converter is simply a coefficient matrix with column-sums equal to unity, in which the rows relate to administrative categories and the columns relate to educational categories. If a vector whose elements are classified educationally is premultiplied by such a matrix, there results a vector whose elements are classified administratively.

## 4. Part-Time and Informal Education

Part-time formal education plays an important role in all educational systems. Much of it is the product of private tuition, but until some means of documenting this has been found we must be content with recording only part-time education taking place in educational establishments.

This again suggests an institutional classification as the starting point, with eventual subdivisions reflecting subjects studied and tests passed. But the integration of these details into a recording system is likely to be more difficult than with full-time education because of the greater variety of arrangements possible. A certain proportion of the people attending part-time courses will be economically inactive: the housewife, for instance,

who while her children are at school devotes some of her time to social studies or to learning a craft. But the majority are likely to be economically active, and the institution to which they will be attached on a more or less permanent basis will be their work-place. They may also be attached for a number of years to an establishment of formal education as part-time students, in which case they are, in principle, no more difficult to record than full-time students: all that need be done is to divide the economically active into two categories, those who attend an educational establishment on a part-time basis and those who do not. But in other cases their part-time education may take the form of a single short course or a succession of short courses taken at different establishments. Such people are not on the books of any educational establishment for long and may be on the books of more than one establishment concurrently; collecting information about them will not be easy. In English labour statistics the economically active population is classified by industry and sometimes by occupation, but rarely, if ever, by part-time educational activity. Educational statistics may provide an industrial classification for some categories of part-time students, such as employees attending part-time courses on day-release schemes, but this is only a small part of the picture.

For informal education the classification by educational establishment does not apply. A possible alternative might be a classification by qualification aimed at: in England, a diploma in engineering, say, or fellowship of an institute of accountants. The recording difficulties here are even greater than in the case of part-time education. An aspirant engineer or accountant who decides to prepare himself for the professional examinations of his association may do so in a variety of ways and may or may not be attached to an industry. Up to a point, information could be obtained from the professional examining bodies. Failing this, it would have to be obtained from the individuals themselves.

The conclusion seems to be that if we want to merge the educational history and the work history of people engaged in part-time or informal education we must be prepared to adopt a system of individualized records.



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# VI

## THE ACTIVE SEQUENCE: EARNING ACTIVITIES

### 1. The Organising Principles

As in the case of learning activities, introducing earning activities into the demographic accounts is, formally, a simple matter: given $n$ types of employment to be distinguished, the relevant ages or age-groups are subdivided into $n+1$ categories, the supplementary category being required to accommodate the people who are economically inactive. Earning activities can run parallel with learning activities and interact with them, in which case a single residual category is sufficient for the two classes of activity, to accommodate, together the people who are both educationally and economically inactive.

The age-span to be covered can go from the end of compulsory education to the retiring age or more. As long as learning activities remain important, say up to the age of 25 or 30, it would be desirable to record employment flows for each year of age. Thereafter it would probably be sufficient to work with five-year age-groups.

The core of the concept of employment is full-time employment in a particular workplace, but a means must also be found for handling part-time and spare-time activities.

As to the classification to adopt, the most practicable, at least at the beginning, is undoubtedly an industrial one, corresponding to the classification of learning activities by educational institutions discussed in section 3*a*) of the preceding chapter. The next classification to consider is a classification by occupation, which can be said to correspond to the classification by subject streams discussed in section 3*b*) of the same chapter. Occupational titles may also contain an element of status, analogous to the grades discussed in section 3*c*).

An individual's occupation is likely to be connected with the quality, quantity and up-to-dateness of his education. It is therefore necessary to introduce some interaction between educational qualifications and occupation. Where teaching manpower is concerned the connection is all-important. Indeed, in the case of teachers not only would the classifications suggested above have to be subdivided into more categories than can be contemplated for other sections of the labour force, but it might also be desirable to go further in the interaction of different classifications.

These topics are discussed in order in the following sections.

## 2. The Production Boundary

The earning activities to be recorded can be defined with the help of the concept of the production boundary, which has already been mentioned in connection with the educational boundary. The core of the concept of production is the output of goods and services for the market. The greater part of these goods and services is produced in clearly distinguishable workplaces, or establishments: farms, mines, factories, shops and so on. It has to be recognized, however, that not the whole of the output of every establishment is intended for the market. In many farms, for instance, a certain part of the output is not sold but is used to feed the farmer and his family; since this unmarketed output is produced in a market-oriented establishment, or at least in an establishment which is typically market-oriented, it is usual to include it in the measurement of production. Again, letting living accommodation is one of the services which are considered to be part of production, its valuation being based on the measurement of rent; in the case of owner-occupied dwellings, where the landlord (the producer) and the tenant (the consumer) are one and the same person, it is usual to impute a rentable value to the dwellings, so that the services they provide should not be excluded from the measurement of production.

Where production takes place in an institution or in circumstances which are not market-oriented, the *prima facie* assumption is that it should not be recorded in the economic accounts. This applies particularly to unpaid household activities and amateur activities. Members of households and amateurs produce many goods and render many services to one another, cooking meals, sewing and knitting clothes, decorating rooms, driving children to school, painting pictures, performing on musical instruments and so on, which are not included in production, although they would be if they were sold on the market. All that is normally recorded as production are the goods and services bought by the household, without any imputation for their further processing by the household members themselves. Households, it is said, lie outside the production boundary.

Although this treatment gives rise to such well-known paradoxes as that total output is reduced if a man marries his cook and she continues to cook for him, it is easy to see that some such convention is essential; partly because household and amateur activities are hardly documented at all, so that in making imputations there would be very few data to go on, and partly because they merge imperceptibly into the process of living, which goes far beyond the scope of economic science. It is possible to be shaved every day by a barber, to have one's shoes cleaned by a shoe-shine or to play tennis with a professional; but the study of production, as it is normally understood, would gain nothing from an attempt to include the activities of shaving oneself, cleaning one's own shoes or playing tennis with one's friends.

An intermediate situation arises where the activities of government departments and private non-profit-making institutions are concerned. These activities, although not market-oriented, are well-documented and clearly circumscribed. The services they provide fall easily into classifiable categories, such as education, medical care, road maintenance, defence, each with its own cost structure and financing mechanism. In other words,

they can be measured in economic terms and so it is meaningful to include them in the production boundary.

If these criteria are accepted, productive activity can be defined as: any activity which is directed to the production of goods or services, which is measurable in economic terms and in which, generally speaking, people are gainfully employed. And this definition leads naturally to a definition of earning activity as: any form of gainful employment in productive activity. There is widespread agreement among economists about these definitions, particularly in those countries, including all the member countries of the OECD, which make use of the ideas underlying the United Nations' System of National Accounts (SNA) in integrating their economic statistics. Russia and most other socialist countries make use of the ideas underlying the Material Products System (MPS), which excludes many services from the concept of production, with the consequence that people can be gainfully employed outside the sphere of production. However, thanks to the recent work of the Conference of European Statisticians on links between the two systems, it is now possible to translate anything said in the language of one into the language of the other. For present purposes it is convenient to use the concept of production as it appears in the SNA.

## 3. The Labour Force

Following the argument of the preceding section, it can be said that anybody gainfully employed in a productive activity is a member of the labour force. Before this simple statement can be accepted as a sufficient definition, however, two points must be cleared: does the term "employed" refer only to employees or does it include the self-employed; and how should the unemployed be classified.

Labour statistics usually distinguish between employees and self-employed, the former being given a much more detailed treatment than the latter. The same distinction appears in national economic accounting, where the remuneration of employees takes the form of wages, salaries and various supplements to labour income, while the remuneration of the self-employed is treated as part of the operating surplus of businesses (a form of profit) and is not further differentiated. In demographic accounting these distinctions are irrelevant and the labour force must be understood to include both employees and self-employed, since in tracing the flows of human beings through active life we are as much interested in the farmer or shopkeeper as in the farmworker or shop assistant, in the sculptor as in the navvy.

As to the unemployed, meaning people temporarily out of work, their position is less definite. They can be treated either as a special category of the economically active, that is as part of the labour force, because their intention is to be economically active and this is their normal condition, or as a special category of the inactive. If they are treated as a single undifferentiated group or are classified otherwise than by their last activity, for instance by duration of unemployment, it would seem desirable to include them among the inactive. But if they are classified by type of activity, then it would probably be best to ignore altogether the distinction between employed and unemployed in the recording of flows and make the



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distinction only in the tabulation of stocks. This second treatment would seem particularly appropriate where unemployment is of short duration.

## 4. Full-Time, Part-Time and Spare-Time Employment

In trying to classify the labour force by economic activities we run into precisely the same kind of problems as we met when trying to classify students by educational activities.

As with students, the simple case is that in which an individual works full-time in a single establishment. Making all allowances for the existence of multiple jobs and of part-time and spare-time work, this case is of overwhelming importance and justifies the idea of regarding a particular workplace as the primary centre of activity of most members of the labour force just as, in the preceding chapter, we regarded a particular educational establishment as the primary centre of activity of all full-time pupils and students.

The existence of multiple jobs poses the problem of assigning the multiple-job holder to his principal activity. This can be attempted by reference to his main, continuing centre of activity. Thus the university lecturer in engineering who regularly does a certain amount of consulting work for an engineering firm should be assigned to the educational industry; the stockbroker who owns, and in part runs, a farm should be assigned to the finance industry. In some cases the holding of several jobs is more a matter of words than of reality. The university teacher who is seconded to a government department for a number of years but who is able to resume his university appointment at the end of his period of secondment is not a multiple-job holder; in principle at any rate he passes from one job to another and back again. Company directors who hold no regular office but are required to attend monthly meetings of their board might either be treated as spare-time workers or, if they are highly active in this field, as self-employed workers under the heading of business services.

Part-time workers are those who do some paid work but are primarily attached either to a learning activity or to the residual group of the economically and educationally inactive. In most cases it should not be difficult to decide which is their principal activity: students who work their way through college are primarily students and secondarily workers; workers who spend a day a week attending courses at a technical college are primarily workers and secondarily students. An example of a part-time worker from the educationally and economically inactive group is the wife of a graduate student who spends three mornings a week as an assistant on a research project.

Spare-time workers are those who have less regular and definite commitments than those considered in the last paragraph. Generally speaking, they will have a specific task assigned to them and will be paid for what they complete. An example is the housewife who undertakes to sew or knit at home for a fashion shop on a piece-work basis. Another example is the employee who does odd jobs in his spare time.

Many forms of seasonal and occasional work further complicate the picture. Hop pickers, temporary postmen and shop assistants at Christmas time, waiters who work in a locality during its season but for the rest of the year are differently occupied or are unoccupied, students who take

a job during the summer vacation are all examples of the many mixtures of activities that can be found in the real world.

As in the case of educational activities, the best course is to begin with the simple cases and then try to elaborate the picture as far as it seems important to do so. Apart from the decision to ignore certain issues as uninteresting or unmeasurable, the choice of the period of account will automatically rule out certain complications. For instance, there are no hop pickers of temporary Christmas workers on 1st January.

## 5. The Industrial Classification of the Labour Force

In choosing our classifications we should keep in mind not only their theoretical relevance to our purpose but also their statistical practicability. The choice of an industrial classification as the primary classification for earning activities satisfies both these conditions: it accords with our concept of employment as full-time employment in a particular workplace, since an industry is nothing but an aggregate of workplaces; and it enables us to work with recognized and relatively well-documented statistical entities, since the industrial sector is usually one of the best-documented sectors in any system of national statistics.

Labour statistics as set up at present make it possible to construct fairly detailed age × industry matrices showing the industrial distribution of the labour force in each age-group, the changes in this distribution from year to year and the ages at which members of the labour force enter or leave different industries. These matrices will also show which industries tend to gain labour and which tend to lose it. But they will not show where the gains come from or where the losses go to. In Chapter IV we illustrated the difficulties of making any but a formal link between the flows out of one educational activity and the flows into another. Similar difficulties arise when we try to trace the flows of manpower between economic activities. What we should like to know for each age or age-group in each year is the complete matrix of inter-industry transfers. In practice we are likely to know much less than this. We may reasonably expect to know, by age-groups if not by individual ages, the stock of manpower in each industry at the beginning of each year and the new entrants from educational institutions in the course of the year. We can estimate deaths by age and industry, making some allowance, where necessary, for industrial differentials in mortality rates. This leaves us with the residual inflows and outflows needed to balance the accounts. To a large extent these residual flows are likely to reflect changes in the participation in productive activity of the inactive who take on part-time, spare-time and occasional work, but there are cases where migration may constitute a complicating factor. This difficulty can be met by routing migrants through the accounts for the inactive, except in special cases; for example, where the migrants are known to be students.

## 6. The Occupational Classification of the Labour Force

In tracing the flows of pupils and students through the educational system we saw that it was desirable to distinguish between different academic streams. These streams are distinguished not only by subject but also by the degree to which their content is elementary or advanced.



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Similarly, people engaged in earning activities can be differentiated not only by the industry in which they work but also by the occupation they follow and their level on the occupational ladder. In population censuses it is usual to find extremely detailed occupational tables, but for our purposes it may be sufficient, at least at the outset, to distinguish only broad categories. A first distinction that is useful is between managerial, clerical and technical manpower. A second is the subdivision of technical manpower by degrees of skill: qualified workers, technicians, craftsmen, operatives and unskilled workers.

An attempt can be made to define these five categories of technical manpower either in terms of the capacities expected of them or in terms of the education and training they have typically received. A definition by capacities might run roughly as follows: qualified workers such as scientists, engineers, doctors, barristers and many other professionals, should be expert in their field of knowledge and capable of extending its boundaries; technicians should possess a certain conceptual ability, applying judgment in their work even if they take the techniques they use as given; craftsmen should possess manual and mechanical dexterity and apply judgment in its exercise; operatives, who are mainly machine-minders, should be able to make simple observations and carry out simple manual operations, allowing for the fact that the machinery over which they watch may become increasingly complicated; and unskilled workers need possess none of these capacities since all they are expected to do is fetch and carry or its equivalent. A definition by education and training might run as follows: qualified workers typically have as a minimum qualification either a first degree or membership of the appropriate professional organisation; the minimum qualification for technicians generally involves passing a technical examination; for craftsmen, serving an apprenticeship; for operatives, training on the job; and for unskilled workers, nothing.

These seven broad occupational groups, managers, clerical workers, qualified workers, technicians, craftsmen, operatives and unskilled workers, can be further subdivided by introducing either some finer distinction of skill or some of ranking based, say, on seniority or experience or the degree of responsibility achieved. Such refinements would undoubtedly be very interesting, in fact they would be indispensable if we wanted to make a special study of any productive activity.

There is a secondary classification of technical workers which is important from an economic point of view: the distinction between workers on the main-process of an industry and workers on complementary processes. For example, in the chemical industry the main activity is making chemical products, and a complementary activity is transporting these products in special vehicles operated by chemical companies. Each activity is carried out by different kinds of workers. In the study of productivity, which is generally measured as the ratio of output to labour, it is desirable to distinguish between the two kind of workers. If we look at the main-process workers in any industry we shall typically find that they become more productive with the passage of time: technical and organisational improvements make it possible to economize on chemical workers in the chemical industry and on transport workers in the transport industry. But if a chemical plant decides to set up its own transport service, any reduction in the number of chemical workers employed will be offset by an influx of lorry drivers; and since the industry's output will

normally be measured in terms of the quantity of chemicals produced, irrespective of who undertakes their transport, the ratio of output to labour will be distorted. The correct treatment here is to distinguish between the output of chemicals and the output of transport services by the chemical industry and record separately the labour employed in each process.

The introduction into a system of demographic accounts of occupational classifications would enable us to construct for each year an occupation x industry matrix showing the distribution of the labour force by skill. A succession of such matrices would be useful in the study of technical change and of changes in labour productivity. It would also be useful, in conjunction with output projections, to throw light on the changing demand for different types of skill. All industries, generally speaking, tend to lose unskilled labour and gain skilled labour, but there are many factors which cause systematic transfers between industries, such as the disappearance of agricultural tractor drivers into motor garages or of mathematics teachers into computing firms. If these transfers take place unperceived for several years, they may prove very difficult to correct and may eventually bring about the decline and even extinction of activities which would be worth preserving. In England two cases in point are the scarcity of science teachers in schools and the extinction of the watch industry.

## 7. The Links Between Earning and Learning

In the preceding section an attempt was made to define occupational categories in terms of education and training. In the context of demographic accounting this implies the question: at what points should our earning classifications be made to interact with our learning classifications. More generally, it implies the question: what use does society make of the education it provides. Both these questions can be examined from three angles: the quality of education, the quantity of education and the age-span of education in relation to employment.

### a) *Employment and the quality of education*

Writings on the links between education and manpower often seem to be based on the idea that there is some simple connection between a job and the training necessary to perform it. In some cases this is obviously true, but observation suggests that in others, for instance over a wide range of jobs in the general area of engineering and technology, it is not true at all. Whether this matters or not is at present an open quest on which deserves further study. It has a considerable bearing both on the manpower approach to educational planning and on the issue between generalism and specialism in early education.

From the practical point of view, the interaction of learning and earnings classifications raises a number of problems whose solution depends to a large extent on the data available. In England, for example, while it is often possible, with the statistics that exist, to find out for each age the outflow of leavers from each type of educational institution and the inflow of new entrants into each industry, it is seldom possible to trace the educational outflows to their industrial destinations or the industrial inflows back to their educational sources. A dummy account that collected

all leavers together and reallocated them over industries would provide an accounting link between educational and manpower statistics but not proper integration. What is wanted is a matrix with educational institutions in the rows and industries in the columns or, better still, qualifications in the rows and occupations in the columns. Such qualification x occupation matrices would enable us to see the correspondence between the kind of work for which people are primarily trained and the kind of work on which they actually find themselves engaged. The information needed to construct these matrices for leavers and new entrants should not be too difficult to collect. Eventually, if this principle were followed when recording changes of job, the same treatment could be applied to the whole labour force.

b) *Employment and the quantity of education*

The quantity of education is an interesting and useful concept. Educational systems in different times and places impart widely differing amounts of education to the average individual passing through them, and there are also wide dispersions around these averages. A labour force endowed with more education per head is likely to be more productive than one with less, just as a labour force endowed with more capital per head would be. Indeed, the relationship between output and manpower in different countries is rendered much more uniform if the measure of manpower is adjusted for its average educational content.

The system described in this report is well adapted to providing measures of the quantity of education. These measures can be expressed either in quality, or in time, or in costs at constant prices (that is to say, time weighted by cost), or in discounted expected returns (that is to say, expected gains).

Measures based on quality, that is, on the number and level of qualifications obtained, may seem at first sight the most appropriate. Qualifications are, indeed, the accepted criterion in every-day life for assessing an individual's educational achievement. But if we try to place them on a numerical scale, that is, to establish a meaningful relationship between them, we soon see that they are very difficult to measure: is an A-level certificate worth two O-level ones; a first degree worth three or five or ten A-level certificates; a diploma in accountancy on a par with a diploma in music; and so forth. This is not to say that we should not try to devise a scale for educational qualifications. But until we have succeeded in doing so, we had better be content with measuring the quantity of education as best we can in units we know how to manage.

Measures based on time are the simplest to make and are not without their uses. As we shall see in Chapter X, there are considerable differences in the time spent in the English educational system by groups with distinguishable educational careers; and there can be little doubt that these differences would be greatly increased if a range of countries were brought into the comparison. But since the cost of a year's education varies enormously with the stage of education reached and, at each stage, with the type of education received, measures based on time are for most purposes inferior to measures based on costs.

Measures based on costs are equivalent to the measures of educational "output" used in the national economic accounts, which are actually

measures of economic inputs into education. They are useful, as we shall see in Chapter IX, in that they make it possible to calculate the average educational costs associated with any given state: the costs incurred on average in educating, say, a university graduate in physics. Their drawback is that they assume too simple a relationship between inputs and outputs: they equate the value of education with the expenditure incurred in imparting it. Thus of two boys of the same age in the same form in the same school, one may leave school hardly able to spell and the other may crown his school career at the same time with several O-level certificates; but since certificates cost nothing, the two boys will be deemed to have absorbed the same amount of education. Nevertheless, in the present state of knowledge, measures based on costs, imperfect as they are, are probably the best that can be achieved.

Measures based on expected returns are more difficult to make than measures based on costs. In the first place, it has to be recognized that much, if not most, of the highly educated manpower is absorbed in occupations, principally teaching, where earnings are not determined by a responsive market mechanism but by conventional standards based on tradition. This being so, a large part of those with a long and expensive education may expect to obtain throughout their active life quite mediocre economic rewards. Therefore, if we wish to measure the quantity of education in terms of the economic benefits it is likely to confer on its recipients, we should measure these benefits in those activities which are governed by the market mechanism and impute equivalent earnings to those engaged in equivalent non-market activities. In the second place, if our calculations relate to the future it is unreasonable to base earning differentials on their present relationship unless it can be argued that demands and supplies of skills will move together. For example, a calculation may be made to see whether increased investment in education is justified; the answer will probably be that if the investment is made, the proportion of the population with a high level of education will greatly increase as a consequence, and therefore the earnings of the well-educated will tend to fall in relation to those of the less educated. Thus a basic assumption of the original calculation will be vitiated.

Within the demographic accounts, any of the four methods suggested above would make it possible to measure, year by year, the quantity of education embodied in those who left the educational system that year. If the accounting system had been running over a whole human life-span, it would be possible to aggregate these estimates so as to include in them the whole population. If the accounting system had been introduced only recently, however, information for the older members of the population could be obtained only from a census or a sample survey. Serious difficulties would almost certainly arise over the comparability of standards through time; but quality problems are not new to statisticians even if they are all too often ignored by them.

A final point. Whichever method were adopted, it would be necessary to allow for the quantity of education embodied in the economically inactive part of the population.

c) *Employment and the age-span of education*

Much scientific and technical knowledge has a fairly high rate of obsolescence, and so the contribution it can make to production will tend

to diminish unless it is renewed from time to time. It is this consideration that lies behind suggestions for spreading education more evenly through life instead of concentrating it all in the early years as at present, but so far these suggestions have made little impact on the organisation of formal education.

In many countries compulsory education ends around the age of 15 and retirement begins around the age of 65. This leaves a span of fifty years which can be devoted in part to learning and in part to earning, and an important question is: how should this time be allocated between these two activities. The conventional answer so far has been that, as far as formal education is concerned, learning should be concentrated in the first few years; that in the great majority of cases it would only occupy two or three years, say up to the age of 17 or 18; and that only rarely would it extend beyond the age of 25, becoming more and more specialized as it was prolonged.

This answer implies that technical knowledge and the techniques it engenders change very little, so that knowledge gained in youth will remain useful over a normal working life without much further intellectual effort. This condition is not satisfied in a period like the present, when scientific and technical knowledge is constantly growing. If an individual's capacity to earn depends on his technical ability as judged by the changing standards of the times through which he lives, then he will find it worth while to forego some earnings in order to have the time for technical study; only thus can he maintain his earning capacity. If his early training has emphasized ideas and methods of analysis rather than facts and techniques, he is likely to find it easier to acquire new knowledge later in life, because his view of the world will not be too constricted by his early learning.

The question of the optimal age-distribution of education has been studied by Weizsäcker (in *Mathematical Models in Educational Planning*, OECD, Paris, 1967) on a set of assumptions which, though simplified, bring out its principal features. His main conclusion is that, in addition to periods of full-time learning and full-time earning at the beginning and end of the active life-span, there should almost certainly be a period of part-learning part-earning in the middle range of active life. His second conclusion is that the proportion of total educational effort devoted to this mid-range mixture of activities rises sharply with the rate of obsolescence of technical knowledge. This is so different from the actual educational pattern accepted at present that it would seem highly desirable to see whether these conclusions still hold, or are perhaps even accentuated, when the model is generalized and so made more realistic.

In terms of the demographic accounts, extending the age-span of education would simply mean a longer overlap of the learning and earning classifications and would present no special difficulties.

## 8. Teachers in the Labour Force

Educational workers, especially teachers, call for more detailed treatment than other kinds of worker if our interest is centred on the analysis of educational processes. The problems that arise are always the same. First, at different ages teachers enter the educational system from various sources and leave for various destinations, and sources and destinations

must be linked. Second, if different types of institution are distinguished, it is necessary to account for transfers between them. Third, in addition to age and institution it is desirable to record certain personal characteristics, notably qualifications and general area of specialization.

These points are particularly important if we are looking to the future. From the point of view of demand, we must try to ensure that there are enough teachers with appropriate qualifications and subjects of specialization in the different parts of the educational system. This may not be easy to do in circumstances in which there is an abnormally large number of elderly teachers who will shortly retire, or if the net advantages (pay and other conditions) of teaching are allowed to fall behind the net advantages of other occupations in which potential teachers can be absorbed. There will always be a number of dedicated teachers, just as there will always be a number of dedicated nurses and ministers of religion; but an affluent society would be unwise to rely unduly on the spirit of dedication, a spirit which its commercial driving force sedulously undermines. From the point of view of supply, therefore, it would seem essential to be able to compare the net advantages of teaching with those of other occupations which teachers could follow and to ensure some sort of balance between the two.

If this is not done, teachers may gradually forsake their vocation. In such a situation, the maintenance of the quantity of teachers will be achieved only through a lowering of their quality; the very result which an affluent society would, presumably, wish to avoid.

# VII

## THE PASSIVE SEQUENCE

### 1. The Characteristics of the Sequence

If, as in the last two chapters, we attempt to trace the movements of people through the learning and earning stages of their active life, we find that a great many classifications are involved; and if we were to treat other activities in the same degree of detail, it would be necessary to introduce many more. Numerous as they are, however, the classifications needed to describe the active sequence are only a sub-set of the classifications needed to describe the whole social scene. Let us now turn to another sub-set, which will enable us to describe people at various stages of what we have called in Chapter II their passive sequence; that is to say, which will enable us to characterize them in terms of the socio-economic groupings to which they belong at different periods of their life.

In the passive sequence the basic classification is by household. Most people spend most of their lives attached to a succession of household groups each of which changes its character as time goes on. Some people, for longer or shorter periods and at different times, are primarily attached to what may be called institutional households, such as orphanages, hospitals and almshouses.

As an example, consider a child born into a household consisting of a mother, father and two children in a particular social class and in a particular quantile of the income distribution. Even if for the time being the class and income of the household remain constant, the birth itself changes the category to which the household belongs by changing its size, with the consequence that each existing member of the household moves into a different category: from a four to a five-person household. As the child grows older, the household group to which he belongs is likely to change: it may lose or gain members of all ages; it may change its socio-economic status or its ranking in the income distribution; it may break up in the sense that its central figures, the father and mother, separate or die. In time, the child will grow up and is likely to leave the household of his parents and set up a household of his own. At first he may live by himself or with friends of his own age; later he is likely to get married and the whole process starts again. At any stage a misfortune may occur, disrupting the course of normal life, and the individual may find himself, for a time or indefinitely, the inmate of an institution.

The passive sequence lends itself to the same kind of analysis as the active sequence. From any state in the sequence, observation will show

that individuals tend to move to a limited range of other states in certain proportions. If these proportions remain constant or change in a known way, it is possible to calculate the distribution of states in which an individual born in a given initial state is likely to be found after any given number of years.

Existing statistical arrangements make little or no provision for realizing the passive system in its dynamic form, that is for tracing changes of state from age to age. However, if age is removed we are left with a matrix connecting the states at the beginning of a year with the states at the end of the year. The marginal vectors of this matrix contain nothing but the distribution of the population by type of household, and on this a considerable amount of information is available. Thus the passive sequence, though in itself perhaps a novel concept, is linked by aggregation with a familiar and important body of data which offer a good starting point for its development.

The states distinguished in the passive sequence will depend on the criteria we adopt in classifying households. The first division to be made is between natural households and institutional households. Within these two main types many subdivisions are possible. Let us look at each type in turn.

## 2. Natural Households

Natural households can be characterized in a large number of ways. Size, social class and income have already been mentioned; others are composition, race, religion, location.

### a) *Size and composition*

Size means simply the number of people in the household. However, we should almost certainly wish to distinguish children from adults, separating, for example, a household composed of a married couple and two children from one composed of a mother and three children, or of four adults living together, or of a father, two children and a grand-mother. Such distinctions as these greatly complicate the concept of size.

In addition, there is no obvious definition of a child or an adult. To say that adulthood begins at puberty is a biological irrelevance; to say that it begins at twenty-one is a legal fiction. In contemporary conditions it is perhaps best to take the age after compulsory school attendance, the age at which it is possible to become self-supporting, as the dividing line. In a country where the school-leaving age is fifteen, individuals aged nought to fifteen are children and the rest are adults.

Relationships within the household are also important in some contexts. A man and woman living together with their own children may not be the same thing, from a psychological point of view, as a man and woman living together with adopted children or with one another's children by a previous marriage or liaison. Marital status enters as a further complicating factor.

### b) *Social class*

Since this classification is usually given in addition to a classification by income, it is desirable that it should reflect social rather than economic



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differences. In fact it is usually heavily dependent on the occupation of the head of the household, though attention is sometimes paid also to the form in which income is received and to the size of the income. There is a danger that these criteria may lead to a classification which is closely related to a purely economic one, with the result that social status and economic status appear more closely correlated than they really are. This could be avoided by broadening the range of criteria; by including, for instance, educational characteristics, familiarity with the lives of ancestors and existence of inherited possessions.

c) *Income*

The consolidated income of all the members of a household is in principle easy to define, but the practical difficulties of collection often result in the use of the income of the head of the household as a proxy. However, in view of the importance in many cases of the earnings of the wife of the head and of their grown-up children, it will often happen that the income of the head will not be a good indicator of household income.

d) *Race and religion*

As we all know, these are particularly problematic concepts. People may feel strongly about their race without anyone being able to tell them what it is; and to say that one is a protestant, a catholic, a buddhist or an atheist says very little about one's religious beliefs or moral standards. Indeed, for the numerically dominant group in a society the words race and religion may be almost devoid of meaning; whereas they will be loaded with explosive and perhaps distorted meaning by racial and religious minorities, which are often quite untypical of their race or religion. A more accurate definition might be reached by adding a cultural dimension; by distinguishing, for instance, between American negroes and African negroes, between English catholics and Irish catholics, between buddhist Indians and moslem Indians. Another useful distinction in many communities might be a distinction between established residents and recent immigrants.

e) *Location*

Location may be thought of either in terms of geographic area or of population density, namely of the urban-rural dichotomy. The urban category, in turn, may be subdivided by city size and, within cities, by district of residence. Here again, length of residence may be significant to throw light on certain patterns of behaviour.

3. INSTITUTIONAL HOUSEHOLDS

At any time, many individuals are to be found living not in natural households but in institutions such as orphanages, homes for old people, hospitals, lunatic asylums and prisons. The first distinction to be made here is, obviously, a distinction by purpose. A second distinction might be one between institutions run by private organisations and institutions run by public authorities. A third might be by size. A fourth, given the religious complexion of many charitable bodies, might be by denomination.

In some institutions, such as hospitals, asylums and prisons, the duration of most sojourns is short: to be measured in weeks rather than in years. There arises, therefore, the kind of problem that we have already met in connection with foreign visitors and with the unemployed: should we record states of only short duration? In a study of the institutional population the answer would of course be yes. But if our main concern is with the social pattern as represented by household groups, the answer in most cases would seem to be no: nothing useful would be gained by recording a change of state every time an individual whose living centre is his family entered a hospital for a short treatment. All that seems necessary is to record what may be termed the migrant element of the institutional population, the inmates who are expected to remain inmates for a long time.

There are cases, however, where the distinction between permanent and temporary is equivocal. Boarding schools are an example. In dealing with educational flows in the active sequence it has been assumed implicitly that boarders and day-boys alike are attached primarily to their school throughout the school ages, so that no distinction is made between the two. Conversely, in the passive sequence they might all be classified according to the type of household to which they belong. Yet it may well be that the two groups develop significantly different attitudes to life in certain respects, and that the existence of a "double home" in the case of boarders should be recognized explicitly in the system of passive flows.

## 4. The Links with the Active Sequence

It is a truism to say that family background is an important determinant of behaviour and achievement in life. In terms of our system of demographic accounts, this means that we must introduce a certain amount of interaction between the passive and the active sequences. But since, for practical reasons, there is a limit to the number of categories which we can allow to interact simultaneously, we must first decide on which aspect of the social scene we want to study and then select from each sequence only those categories which will serve our purpose. This can be illustrated by considering the links between the passive sequence and the learning stages of the active sequence.

In the first place there are family size and family composition. Probably the most important factor here is simply the number of children in the family, since there is a well-marked negative correlation between an individual's scores in intelligence tests and the number of brothers and sisters he has, as shown for instance by Maxwell (*The Level and Trend of National Intelligence*, Scottish Council for Research in Education, XLVI, University of London Press, 1961). But other aspects of family composition matter too, though they may not lend themselves to a simple quantitative treatment. Thus the loss of a parent may affect a child's relationship with his school less than the existence of a family situation of which he is ashamed but which he can neither hide nor explain.

In the second place there is social class. The social group to which a child belongs, especially if it is given a cultural rather than an economic or a caste connotation, almost certainly plays a big role in determining his attitude to learning, not so much by affirming or denying the value of

education as by simply taking its importance for granted. Thus a well-educated home is likely to reinforce a child's innate ability, as far as can be judged by scholastic tests, whereas an ignorant home is likely to reinforce a lack of ability. As a consequence, social differences in scholastic performance tend greatly to exceed any social differences there may be in innate ability. It is probably a mistake to suppose that this gap is much widened, if not actually created, by differences in school facilities; the most powerful factor here is probably the degree to which educational values (however they may be interpreted) are unquestioned. Most schools take it for granted that everyone does his best, "plays the game", and children are more likely to act in this way if they feel the same attitude at home; but if these assumptions are questioned at home, reasons can be found for any form of aberrant behaviour at school.

In the third place there is income. In a country in which public education up to school-leaving age is free, and all manner of grants and awards are available for those who pass the admission tests to institutions of higher education, it is hard to believe that income *per se,* as distinct from other family characteristics, can be of much importance. Any correlation found between scholastic achievement and family income is probably due in a large measure to the correlation between income and social or cultural characteristics. But more research is needed to disentangle the factors at work.

In the fourth place there are race and religion. As was suggested above, the importance of these two concepts differs widely from country to country and from one community to another. In those cases where racial or religious differences are stressed enough to influence behaviour, these differences should be recognized explicitly. In other cases a more telling distinction, from an educational point of view, might be the distinction between the children of established residents and those of recent immigrants, since the latter are likely to find the ways of their new country unfamiliar, the language hard to master and the educational methods bewildering, and their performance at school will suffer accordingly.

Finally, there is location. In a country with some measure of local autonomy in educational matters, regional differences are likely to have developed historically, largely as a consequence of educational experiments; but in a homogeneous, adaptive society good ideas have a chance of being imitated and bad ideas abandoned throughout the country. In a country where the organisation of education is wholly centralized, regional differences will be even more attenuated. In both cases, population density is likely to matter more than region: not only is it more difficult to provide educational facilities in sparsely settled areas, but longer travel times are involved and the force of tradition in favour of early school-leaving may well be stronger. Thus in a period of rapid educational expansion like the present, it can be expected that all areas will follow the trend but that the towns will lead the way.

It is not easy to summarize this discussion and quite impossible to be dogmatic in the present state of knowledge. Given that one should avoid an excessive multiplication of categories, it would seem that a classification of households by number of children, educational level of the parents and urban-rural location would perhaps be the most illuminating at this stage. Some very interesting work on this subject has been done by Freytag and Weizsäcker (*Schulwahl und Schulsystem in Baden-*

*Württemberg,* Heidelberg, 1968) and by Douglas and others (*All Our Future,* Peter Davies, London, 1968).

## 5. "The Great Web"

The links between the passive and the active sequences have been exemplified in the preceding section by a discussion of the relationship between family background and education. One could analyse in a similar way the relationship between family background and occupation or any of the other categories in the active sequence. Furthermore, as suggested in Chapter II, other sequences, such as a medical sequence and a sequence of delinquency, could be developed independently and then linked with the others at selected points to throw light on specific issues.

The accounting matrices for the four sequences, the active, the passive, the medical and the criminal, would each contain the whole population classified in different ways; every individual, therefore, would appear in all four matrices. If the information used in filling in the matrices came from an individualized data system, we could take any element of any matrix and classify its component individuals according to their position in the other matrices. This is equivalent to saying that we could superimpose the classifications of one sequence on those of the others; and in this situation there would hardly be any point in speaking of four sequences, since they would simply represent different consolidations of a single more complex sequence.

The reason why four separate sequences are proposed in this report is that they represent different ways of partially integrating data on human stocks and flows, each way relating to an important aspect of life and not necessarily requiring individualized data for its realization. This last consideration is of some importance at the present time since most countries do not use registration numbers at all and the rest use them only over a limited range.

To sum up. The approach followed here in trying to integrate demographic and social statistics is to make use as far as possible of known concepts and methods of collection and then to show how, in statistically ideal circumstances, the resulting subsystems could themselves be integrated into a single ideal system. The alternative approach consists of trying first to envisage the ideal, all-embracing system and then to show how it can be reduced to something that can be realized in practice. It is usually claimed for the first approach that it is desirable to work out from what we know and that we shall be able to formulate an ideal system only when we have continued this process far beyond its initial stage. It is usually claimed for the alternative approach that it is desirable to try to formulate an ideal system at the outset because it may save us running into too many blind alleys through excessive preoccupation with existing concepts and methods. Evidently there is something to be said for each point of view.

# VIII

## LEARNING ACTIVITIES AND THE ECONOMIC ACCOUNTS

### 1. The Costs of Education

In Chapter V we discussed the problems of classifying the flows of pupils and students into, through and out of the educational system and in Chapter VII we considered the social characteristics which are likely to influence the attitudes to and opportunities for education. There is another important aspect which must be taken into account in any study of the educational system, and that is the economic aspect. Without going so far as to say that every citizen should, or indeed could, have a Ph.D., it must be recognized that educational facilities, even in the most advanced countries, are not in line with the learning capacities of the population. But providing these facilities is very expensive and means sacrificing other social and personal advantages, since neither the supply of money nor the supply of manpower is unlimited. If we are interested in educational policy and educational planning, therefore, it is not enough to look at the transition proportions obtained from our demographic matrices; these will only help us to estimate the future demand for education. We must also measure the costs of education, placing them in the wider context of the national economic accounts. Then, by combining our economic measures with our demographic measures, we can see what need be done to ensure that the supply matches the demand.

The next section describes how to set up the detailed economic accounts of education within the framework of the SNA and the succeeding section gives a numerical example drawn from British statistics for 1965. The chapter ends with a discussion of the links between the economic and the demographic accounts of education.

### 2. Education in the National Accounts

The SNA distinguishes between industrial activities proper, which are directed to the production of goods and services for the market, and the productive activities of private non-profit institutions and government departments. This dichotomy enables a distinction to be drawn between market-oriented and non-market-oriented productive activity; but the definition of the categories in these two broad divisions of productive activity is such that each category can be assigned to one or another of the headings used in the International Standard Industrial Classification

of all economic activities. In this classification, industries are defined in terms of their principal products, such as food, textiles, chemicals, transport, etc., and this criterion is extended to the definition of non-market activities; for example, public administration is defined as producing administrative services, education as producing educational services, and so on.

In the SNA, all producing units are provided with two types of account, an account for their productive activity and an account for their capital formation. They are not provided with balance sheets, though reference is made to the usefulness of statistics of the gross stock of tangible assets employed in different activities.

Financing units too are provided with two types of account: an income and outlay account and a capital finance account. Unlike producing units, they are also provided with balance sheets, the entries in which are linked through time partly by the capital finance accounts and partly by revaluations. In some cases, however, these financial accounts are not sufficient to trace financing flows from their sources to their ultimate destinations, and dummy accounts must be set up to supply the missing links. The most important in this connection are the so-called purpose accounts, described in detail below.

Educational organisations are treated primarily as producing units and divided into three classes:

*i*) private profit-making enterprises;
*ii*) private non-profit institutions;
*iii*) departments of central and local government.

Like all other producing units, they are provided with activity (or production) accounts and capital expenditure accounts. In addition, classes *ii*) and *iii*) are provided with purpose accounts. The functions of these three types of account are as follows.

a) *Activity (production) accounts*

These accounts show, on the outgoing side, the costs of all the inputs, both intermediate and primary, required to produce a given output, in this case educational services; and on the incoming side, the revenues which enable these costs to be met, whether they be revenues from the sales of the output or income from transfers.

Activity accounts are needed for each of the three classes mentioned above. Within each class various kinds of institution, such as primary schools, secondary schools, teacher training colleges, colleges of further education and so on, are distinguished, each with its own account. In the case of institutions run by public authorities, separate accounts are set up for administration and inspection and for special services such as school meals and school health.

For the institutions in class *i*), revenue takes the form of fees. Expenditure is allocated between purchases of commodities (intermediate inputs), compensation of employees (preferably subdivided between teaching staff and non-teaching staff), and other components of value added.

For the institutions in classes *ii*) and *iii*), fees and similar contributions paid by pupils and students or their parents usually play a minor role and the greater part of revenue comes from the corresponding purpose accounts. Expenditure is allocated as in class *i*).

b) *Purpose accounts*

These accounts, which are needed only for classes *ii*) and *iii*), provide a link between the activities of private non-profit institutions and government departments and the various forms of transfer which constitute their main source of revenue. Within each class, purposes and activities are in one-to-one correspondence with one another. The difference between them is that while activities indicate an area of operation, purposes indicate an area of concern: the government's financial contribution to secondary education, say, as opposed to the running of state secondary schools. Each purpose account collects all the transfers intended for current expenditure on a particular purpose and redistributes them so as to further that purpose: either by financing directly the corresponding activity or by giving grants and other benefits to institutions or individuals connected with that activity.

For the purposes in class *ii*), revenue can take three forms: income from the institutions' own endowments; voluntary contributions from the private sector; and public grants. Expenditure is almost wholly devoted to financing the corresponding private activity, except for a small part which goes in scholarships to individuals.

For the purposes in class *iii*), revenue consists simply of grants received from public authorities. Expenditure may be devoted either to financing the corresponding activities, both public and private, or to giving grants to individuals. Thus a distinction is made between public grants to institutions and public grants to individuals. There is an arbitrary element here in so far as grants to individuals are intended to cover fees to institutions, since these fees are in many cases paid directly by the financing authority to the institution and the individual receives only that part of the grant which is intended to cover living expenses. Nevertheless it seems desirable that as far as possible all fees, however financed, should appear under one heading, namely as payments by individuals for a service. This can be achieved by showing the full value of the grants as paid by the purpose accounts to the individuals themselves, and then routing that part which is intended to cover fees through the income and outlay accounts of households, whence it flows into the activity accounts of the relevant institutions.

c) *Capital expenditure accounts*

These accounts are needed for all three classes. They receive money from one or more of the capital finance accounts, which in most cases will relate to some larger institution, such as the central government, that has made itself responsible for the capital expenditure of its educational establishments. This expenditure consists of investment in buildings and other capital equipment undertaken to maintain or increase the capacity of the establishments or to improve the facilities they offer.

Income and outlay and capital finance accounts have not figured in the above list. The reason is that educational institutions, with the exception of the profit-making enterprises, have hardly any financial autonomy. They are usually exempt from most forms of taxation and do little saving or dissaving. They spend what money is available to them on their services, and are dependent for their capital expenditure either on the public authorities or on special appeals. The profit-making enterprises



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are usually responsible for only a small part, and not the more interesting part, of the total educational effort. Hence, having established the purpose accounts to take care of the allocation of financial flows, it is justifiable, from a purely educational point of view, to merge the income and outlay and the capital finance accounts of education in larger institutional aggregates.

## 3. A Numerical Example

A numerical example of an educational accounting system drawn up on the above lines is set out in Tables VIII.1 and VIII.2. This example relates to England and Wales in 1965, is based on accounts for the financial years 1964-65 and 1965-66 and covers expenditure on both sexes.

In several respects the example is not quite consistent with what has been said in the preceding section. In the first place, the tables relate only to public authorities and private non-profit institutions, because there are no readily available data on profit-making enterprises. In the second place, the information, even within those two classes, is incomplete: for instance, no details are given on school transport in the capital expenditure accounts of public authorities, the activity and purpose accounts of private non-profit primary schools are missing altogether; and the information on private non-profit secondary schools is confined to the so-called public schools and to direct-grant grammar schools. In the third place, the coverage is far from uniform; for instance, in the public sector the subsidiary services are provided with separate accounts, whereas in the private sector they do not appear at all and it is impossible to tell how far they contribute to total costs.

Because of these gaps and inconsistencies, Tables VIII.1 and VIII.2 should not be taken as a faithful illustration of the accounting treatment proposed in the preceding section but should be accepted as they are intended, namely as a preliminary outline for the lay-out of the economic accounts of education.

### a) *The educational accounts of public authorities*

These accounts are set out in Table VIII.1. Following the order of the SNA, the upper panel of the table contains the activity accounts, the middle panel contains the purpose accounts and the lower panel contains the capital expenditure accounts. Thirteen activities and purposes are distinguished, which can be grouped under the three main headings proposed in the SNA: general administration (column 1); educational institutions (columns 2 through 9); and subsidiary services (columns 10 through 13). The activity accounts in columns 7 and 9 are empty because in Britain public authorities do not operate either universities or institutions of adult education; and the corresponding capital expenditure accounts are necessarily empty too. As was said above, there are no data for capital expenditure on school transport. and so this service is treated as if it simply consisted of buying general transport facilities.

If we look at the incomings into the activity accounts, shown in the first two rows of panel (*a*), we can see that fees and contributions from individuals are negligible except for the parental contribution to school meals, which amounts to about one-third of the total cost of this service.

Apart from this contribution and a small amount of fees in respect of teacher training, the whole cost of these activities is paid for by the public authorities themselves.

The outgoings are shown in the lower part of panel (*a*). Row 3 shows the expenditure on commodities, that is to say on the products of industry in the widest possible sense. In the SNA there is a detailed classification of commodities in terms of the characteristic products of the various industries distinguished in the system; and further, the value of these commodities, expressed at producers' prices, is divided between basic values and commodity taxes. Although these distinctions are not illustrated in this example, the differentiation of commodities is of course essential for the purposes of input-output analysis and the separation of commodity taxes from basic values is highly desirable.

More attention is paid here to what the SNA calls "compensation of employees", which in this group of activities constitutes the main component of value added. In rows 4, 5 and 6, it is divided into three parts: salaries and employers' contributions to superannuation schemes and national insurance in respect of teachers; wages, salaries and contributions in respect of other staff; and items of labour income, in all cases very small, that cannot be classified under either of these headings. Labour inputs are of great importance in all government activities: in the present case, teachers account for nearly half of the total cost; and all forms of labour, for nearly two-thirds.

The last row of panel (*a*), row 7, relates to loan charges. These charges, mainly interest, are incurred largely in respect of past building programmes and can be interpreted as a crude measure of imputed rent, a measure which could undoubtedly be improved on. Actually, in the SNA no imputation is made in respect of government buildings, and at the national level this decision can be defended on the grounds that the amounts involved are small in relation to the main national aggregates and that the figures are not much used for policy purposes. At a more detailed level, however, these arguments lose their force, since, in forward budgeting, provision should be made for all anticipated future costs, particularly when changes in activity levels are being contemplated.

The incomings into the purpose accounts of public authorities are shown in the first two rows of panel (*b*), divided between receipts from the central government and receipts from local authorities. In this example the division of the receipts depends on the authority responsible for making the payments and so does not reflect the share of the financial burden borne by the two branches of government.

The outgoings take four main forms: transfers to the corresponding government activity, row 10, equal to row 2 in panel (*a*); transfers to the corresponding private non-profit purpose, row 11, which is equal to row 9 of Table VIII.2 below; a variety of transfers to individuals, rows 12 through 19; and international subscriptions, row 20. The transfers to individuals partly find their way back to the finance of educational activities in that they help to pay the fees of private non-profit institutions shown in row 1 of Table VIII.2. There is, however, no definite correspondence between the two sets of figures: transfers may be intended to cover ordinary living expenses as well as strictly educational expenses; and many fees and contributions paid by individuals are not reimbursed to them by public authorities.

TABLE VIII.1. THE EDUCATIONA
ENGLAND AN

| | General administration | Nursery schools | Primary schools |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1. Fees and contributions from individuals | | | |
| 2. Transfers from corresponding government purposes | 46.5 | 3.3 | 305.6 |
| Total incomings | 46.5 | 3.3 | 305.6 |
| 3. Purchases of commodities (current) | 18.7 | 0.7 | 54.3 |
| 4. Salaries and employers' contributions: teaching staff | 1.2 | 1.3 | 204.0 |
| 5. Salaries, wages and employers' contributions: others | 26.0 | 1.2 | 22.9 |
| 6. Other labour income | 0.1 | | 0.4 |
| 7. Loan charges | 0.5 | 0.1 | 24.0 |
| Total outgoings | 46.5 | 3.3 | 305.6 |
| 8. Receipts from central government | 5.7 | | |
| 9. Receipts from local authorities | 41.7 | 3.3 | 306.2 |
| Total incomings | 47.4 | 3.3 | 306.2 |
| 10. Transfers to corresponding government activities | 46.5 | 3.3 | 305.6 |
| 11. Transfers to corresponding non-profit purposes | | | |
| 12. Fees for pupils at independent and direct-grant schools | | | 0.2 |
| 13. Fees for pupils receiving special education | 0.2 | | |
| 14. Maintenance allowances for pupils over school-leaving age | | | |
| 15. Teacher training awards | | | |
| 16. University awards | 0.2 | | |
| 17. Further education awards for degree or equivalent courses | | | |
| 18. Further education awards for other courses | | | |
| 19. Other awards and grants | | | 0.4 |
| 20. International subscriptions | 0.5 | | |
| Total outgoings | 47.4 | 3.3 | 306.2 |
| 21. Finance from central government | | | |
| 22. Finance from local authorities | 1.1 | 0.2 | 37.4 |
| Total incomings | 1.1 | 0.2 | 37.4 |
| 23. Purchases of commodities (capital) | 1.1 | 0.2 | 37.4 |
| Total outgoings | 1.1 | 0.2 | 37.4 |

CCOUNTS OF PUBLIC AUTHORITIES
/ALES, 1965

£ *million.*

| Second-ary schools | Special schools | Teacher training colleges | Univer-sities | Other insti-tutions of further educa-tion | Insti-tutions of adult educa-tion | School meals and milk | School health | School transport | Recrea-tion and training | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | |
| | | *a)* ACTIVITY (PRODUCTION) ACCOUNTS | | | | | | | | |
| | | 0.5 | | | | 38.1 | | | | 38.6 |
| 375.2 | 20.5 | 28.2 | | 136.3 | | 76.8 | 17.9 | 15.7 | 12.5 | 1,038.5 |
| 375.2 | 20.5 | 28.7 | | 136.3 | | 114.9 | 17.9 | 15.7 | 12.5 | 1,077.1 |
| 71.1 | 5.1 | 8.8 | | 20.2 | | 67.6 | 6.9 | 15.7 | 6.3 | 275.4 |
| 226.3 | 9.0 | 11.2 | | 81.9 | | | 0.2 | | 0.7 | 535.8 |
| 24.3 | 4.0 | 4.5 | | 16.4 | | 43.5 | 10.4 | | 4.3 | 157.5 |
| 0.4 | 0.1 | 0.1 | | 0.2 | | | 0.1 | | | 1.4 |
| 53.1 | 2.3 | 4.1 | | 17.6 | | 3.8 | 0.3 | | 1.2 | 107.0 |
| 375.2 | 20.5 | 28.7 | | 136.3 | | 114.9 | 17.9 | 15.7 | 12.5 | 1,077.1 |
| | | *b)* PURPOSE ACCOUNTS | | | | | | | | |
| 7.2 | | 11.1 | 96.8 | 2.2 | 1.0 | 76.8 | | | | 200.8 |
| 387.7 | 27.4 | 39.6 | 33.2 | 156.4 | 0.2 | | 17.9 | 15.7 | 12.5 | 1,041.8 |
| 394.9 | 27.4 | 50.7 | 130.0 | 158.6 | 1.2 | 76.8 | 17.9 | 15.7 | 12.5 | 1,242.6 |
| 375.2 | 20.5 | 28.2 | | 136.3 | | 76.8 | 17.9 | 15.7 | 12.5 | 1,038.5 |
| 6.8 | | 11.5 | 94.3 | 2.2 | 1.2 | | | | | 116.0 |
| 10.8 | | | | 0.1 | | | | | | 11.1 |
| | 6.6 | | | | | | | | | 6.8 |
| 1.1 | | | | 0.1 | | | | | | 1.2 |
| | | 10.7 | | | | | | | | 10.7 |
| | | | 35.7 | | | | | | | 35.9 |
| | | | | 8.7 | | | | | | 8.7 |
| | | | | 10.8 | | | | | | 10.8 |
| 1.0 | 0.3 | 0.3 | | 0.4 | | | | | | 2.4 |
| | | | | | | | | | | 0.5 |
| 394.9 | 27.4 | 50.7 | 130.0 | 158.6 | 1.2 | 76.8 | 17.9 | 15.7 | 12.5 | 1,242.6 |
| | | *c)* CAPITAL EXPENDITURE ACCOUNTS | | | | | | | | |
| | | | | | | | | | 2.4 | 2.4 |
| 61.0 | 3.6 | 7.2 | | 28.9 | | 9.9 | 0.3 | | 3.4 | 152.9 |
| 61.0 | 3.6 | 7.2 | | 28.9 | | 9.9 | 0.3 | | 5.8 | 155.3 |
| 61.0 | 3.6 | 7.2 | | 28.9 | | 9.9 | 0.3 | | 5.8 | 155.3 |
| 61.0 | 3.6 | 7.2 | | 28.9 | | 9.9 | 0.3 | | 5.8 | 155.3 |

Panel (*c*) of Table VIII.1 shows the capital expenditure undertaken in respect of each of the activities distinguished in panel (*a*). In this example the treatment is highly simplified: on the outgoings side there is no subdivision by type of capital good nor any assignment of goods to commodity groups; and on the incomings side it is assumed that the educational activities of public authorities have no financial independence and that the sums they receive on capital account from these authorities correspond exactly to their capital expenditures. In fact these activities are provided on current account with more money than they use to meet their current expenses, with the result that some capital expenditure from revenue takes place: thus of the total of £152.9 million shown in row 22, as much as £21.5 million represent expenditure out of revenue as opposed to expenditure out of loans.

It would be a simple matter to show this distinction in the system by: raising the entries in rows 2, 9 and 10 by the amount of capital expenditure out of revenue; adding after row 7 a new row, called, say, current surplus, to contain the sums in question; subtracting the corresponding amounts from row 22 and moving them to a new row above row 21, called, say, transfers of current surplus. This arrangement would not be justified if the transfers are merely a means of correcting budgeting errors, but would be desirable if individual activities are empowered to decide on the allocation of part of their current revenue to capital expenditure.

b) *The educational accounts of private non-profit institutions*

These accounts are set out in Table VIII.2. The estimates in this table are built up from a number of sources and are not as complete as those for public authorities. For example, nothing is known about nursery schools, and information on primary schools is available only in respect of their capital expenditure. Moreover, the subsidiary services, such as general administration and the provision of school meals, are not separately institutionalized: there are only seven headings, relating exclusively to teaching institutions. As a consequence these accounts are not fully comparable with those for public authorities.

Nor are they comparable with each other, because their content reflects different accounting conventions in different institutions. Thus the figures for secondary schools, which in the private sector cover a large number of boarding establishments, are not restricted to tuition costs but include board and lodging as well; on the other hand, the university accounts ignore board and lodging expenses, even in residential universities, and are restricted to research and tuition costs, including the tuition costs incurred by the colleges of Oxford and Cambridge (which are not, administratively speaking, university institutions). In the case of secondary schools the separation could only be made through an application of the methods of cost accounting. In the case of universities it would, in principle, be a simple matter to add accounts for board and lodging in colleges, halls of residence and hostels as a subsidiary service. Of course many university students do not live on university or college premises and are not wholly dependent on these establishments for their meals; their living expenses would seem to have no place in the educational accounts, any more than would the living expenses of schoolchildren who live at home. But where board and lodging are provided by an educational

institution, it would seem desirable to show this separately as a subsidiary service.

The incomings into the activity accounts of private non-profit institutions are shown on the left-hand side of panel (*a*). As can be seen from row 1, the fees and contributions from individuals constitute in this case quite a substantial source of revenue, amounting to nearly one-third of the total. A new item, sales of research services, appears in row 3. The function of this item is to distinguish contract research from research financed by grants, whether public or private.

The outgoings appear on the right-hand side of panel (*a*). In comparison with the public sector, purchases of commodities are relatively important: partly because of the universities, and in particular of their scientific side; and partly because of the much larger number of boarding schools. These factors are also reflected in expenditure on non-teaching staff.

On the incomings side of the purpose accounts, panel (*b*), some new sources of revenue are distinguished. The greater part of this revenue comes in the form of transfers from the corresponding government purposes. Also, allowance is made for the possibility that private purposes may make transfers among each other: in this example, universities help to finance adult education. Grants for research from the central government are shown separately, since they may not be geared at all to the government programme of general support for the universities. All other sources of revenue, including research grants from foundations, are shown in row 12; undoubtedly, some further subdivision would be desirable here.

On the outgoings side, it is to be expected, in general, that some transfers to individuals might make their appearance since a good number of scholarships, prizes and awards are given by private non-profit institutions, even if the main responsibility for this kind of outlay rests with public authorities. But information on this subject is not readily available.

On the incomings side of the capital expenditure accounts, panel (*c*), a distinction is made between capital grants and loans from the central government. Row 17 shows that a significant role is played by private finance. This role would be somewhat increased if the expenditure on building incurred by the colleges of Oxford and Cambridge were included in the accounts. As in Table VIII.1, there is only a single item on the outgoings side of this panel.

## 4. The Links between Economic and Demographic Accounting

Linking the economic accounts of education with the demographic accounts can be a useful aid to educational planning. Let us consider, as a simple example, one of the categories in which all the pupils are engaged in full-time formal education, say secondary modern schools. The activity level of this group of schools can be measured in terms of the number of pupils attending them, and this number can be estimated by summing over the relevant ages in the appropriate school category in the demographic matrix. If we then divide each element in the input structure of secondary modern schools by this number, we obtain a set of input-output coefficients measured in base-year prices per head, and from these we can calculate the average annual cost of secondary-modern education per pupil. As a first approximation, this average cost could be attributed to every state in the secondary-modern range. If this exercise were repeated for each type of

TABLE VIII.2. THE EDUCATIONA
ENGLAND AN

| | Primary schools | Secondary schools | Special schools | Teacher training colleges | Universities | Other institutions of further education | Institutions of adult education | Total |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| *a)* ACTIVI | | | | | | | | |
| 1. Fees and contributions from individuals | | 46.9 | 4.5 | 0.4 | 9.9 | 0.7 | 0.3 | 62.7 |
| 2. Transfers from corresponding private non-profit purposes | | 9.3 | 0.5 | 11.9 | 108.3 | 2.5 | 1.8 | 134.3 |
| 3. Sales of research services | | | | | 1.9 | | | 1.9 |
| Total incomings | | 56.2 | 5.0 | 12.3 | 120.1 | 3.2 | 2.1 | 198.9 |
| *b)* PURPO | | | | | | | | |
| 9. Transfers from corresponding government purposes | | 6.8 | | 11.5 | 94.3 | 2.2 | 1.2 | 116.0 |
| 10. Transfers from other private non-profit purposes | | | | | | | 0.6 | 0.6 |
| 11. Grants for research from central government | | | | | 8.0 | | | 8.0 |
| 12. Endowment income, etc. | | 2.5 | 0.5 | 0.4 | 6.6 | 0.3 | | 10.3 |
| Total incomings | | 9.3 | 0.5 | 11.9 | 108.9 | 2.5 | 1.8 | 134.9 |
| *c)* CAPIT | | | | | | | | |
| 15. Capital grants from central government | 4.0 | 7.0 | 0.4 | 2.8 | 71.8 | 0.6 | | 86.6 |
| 16. Loans from central government | 0.4 | 1.1 | | | | | | 1.5 |
| 17. Finance from other sources | | 8.4 | 0.1 | | 6.5 | 0.2 | | 15.2 |
| Total incomings | 4.4 | 16.5 | 0.5 | 2.8 | 78.3 | 0.8 | | 103. |

educational institution, and if all pupils were engaged in full-time formal education, we could in this way attach a set of unit costs to every educational activity.

It is impossible to give here a numerical example of such calculations because the cost structures shown in Tables VIII.1 and VIII.2 cannot be linked, even partially, with the demographic framework described in Chapter V above and illustrated in Table X.1 below. Apart from the gaps and inconsistencies mentioned in the preceding section, there are two other major stumbling blocks. One is that in the economic accounts no sex distinction is made, although it is a well-known fact that the pattern, and hence the cost structure, of education are not the same for girls as for boys: for instance, fewer girls than boys enter higher education, and of the total

CCOUNTS OF PRIVATE NON-PROFIT INSTITUTIONS
ALES, 1965

*£ million.*

| | Primary schools | Secondary schools | Special schools | Teacher training colleges | Universities | Other institutions of further education | Institutions of adult education | Total |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| RODUCTION) ACCOUNTS | | | | | | | | |
| 4. Purchases of commodities (current).... | | 18.1 | 2.0 | 5.3 | 54.3 | 1.4 | 0.6 | 81.7 |
| 5. Salaries and employers' contributions: teaching staff ..... | | 26.2 | 1.5 | 5.0 | 49.2 | 1.0 | 1.5 | 84.4 |
| 6. Salaries, wages and employers' contributions: others ..... | | 10.8 | 1.5 | 2.0 | 16.0 | 0.8 | | 31.1 |
| 7. Other labour income ..... | | | | | | | | |
| 8. Loan charges ..... | | 1.1 | | | 0.6 | | | 1.7 |
| Total outgoings ..... | | 56.2 | 5.0 | 12.3 | 120.1 | 3.2 | 2.1 | 198.9 |
| COUNTS | | | | | | | | |
| 3. Transfers to corresponding private non-profit activities ..... | | 9.3 | 0.5 | 11.9 | 108.3 | 2.5 | 1.8 | 134.3 |
| 4. Transfers to other private non-profit purposes ..... | | | | | 0.6 | | | 0.6 |
| Total outgoings ..... | | 9.3 | 0.5 | 11.9 | 108.9 | 2.5 | 1.8 | 134.9 |
| PENDITURE ACCOUNTS | | | | | | | | |
| 3. Purchases of commodities (capital)..... | 4.4 | 16.5 | 0.5 | 2.8 | 78.3 | 0.8 | | 103.3 |
| Total outgoings ..... | 4.4 | 16.5 | 0.5 | 2.8 | 78.3 | 0.8 | | 103.3 |

number of young people who do, a larger proportion of girls than of boys goes to teacher training college. The other is that the classification of the economic accounts is a strictly administrative one, and the data available make it impossible to distinguish between different types of establishment within the main institutional groups: to distinguish, for instance, between secondary modern schools and grammar schools; yet both the quantity and the composition of the economic inputs is bound to vary considerably from one type of establishment to another.

Assuming that these difficulties were smoothed out, there would still be a number of problems left. In the first place, if we want our estimates to be meaningful we should distinguish not only between different types of establishment but also between different states: obviously, a senior

pupil studying science in a secondary school costs more to educate than does a junior pupil engaged in general studies. Ideally, in order to get an accurate measure of unit costs we need input data for each age (or grade) and for each subject stream.

In the second place, we must remember that the unit costs of part-time education are likely to be different from those of full-time education. Therefore, when drawing up the accounts of institutions that provide both, we should establish separate cost structures or, alternatively, full-time equivalencies for different categories of students. For example, the students enrolled at establishments of further education in England and Wales are measured in full-time equivalents, counting a part-time day-student as one-fifth and an evening student as one-tenth of a full-time student.

In the third place, we should try to devise a rational treatment for subsidiary services. Since they are for many children and young people an indispensable adjunct to education, we should not ignore them. But since they are not used by all, we should keep them separate from the educational activities proper and, when calculating unit costs, we should take care to relate them only to that proportion of the student population which uses them. Also, subsidiary services are in most cases independent of the type of education they happen to accompany: it costs no more to feed a student of physics than to feed a student of English. This consideration applies equally well to administration: the headmaster of a grammar school of a given size does not need more secretaries than does the headmaster of a secondary modern school of the same size. These ancillary activities, therefore, require different grouping criteria: the cost of the school meals service should be divided by the total number of children who avail themselves of it, irrespective of age, grade or type of institution; more widely still, the cost of administration and inspection should be divided by the number attending all types of school, both state-run and private, with the possible exception of unrecognized private schools which are not inspected by the educational authorities.

Incidentally, what has been said so far has implied the use of money values to measure inputs, because this is how inputs are usually measured in the economic accounts; but many of them can be measured in other ways. Thus teachers can be measured by number, classrooms by area, ink in gallons and so on.

Once we have got our measurements of unit costs, or inputs per head, straight, we can combine them with the demographic accounts to make a variety of calculations about the future. For example, we can work out from the demographic accounts through what educational sequences the people found this year in a given state must, on average, have proceeded in the past in order to reach their present state; and from this we can calculate the total past costs associated, on average, with that state. In other words, we can calculate the amount of educational inputs absorbed in transforming a baby into, say, a twelve-year-old grammar-school boy. Conversely, we can work out from the demographic accounts how the people found this year in a given state are likely, on average, to proceed in future years through the educational sequences open to them; and from this we can calculate the total future costs associated, on average, with that state. In other words, we can calculate the amount of educational inputs which a twelve-year-old boy at grammar school is likely to absorb to complete his education. From this it follows that if we can estimate the

number of twelve-year-old boys at grammar school in, say, 1980, we can also estimate the total resources needed to educate them up to and beyond that age. And the same can be done for all the states in the system. Methods for making these calculations are described in Chapter IX, and some of the results that can be obtained by applying them to real data are given in Chapter X.

The above reasoning assumes, of course, that both the pattern of demand for education and the input structure of education remain constant through time. In fact both these groups of relationships change, but not so fast as to invalidate the usefulness of the projections suggested above. Provided we do not set our sights too far into the future, these projections can give us a fairly good idea of the orders of magnitude involved. If we want to refine our estimates, however, or project them further into the future, the assumption of fixed coefficients must be abandoned and new problems arise.

Observed input-output coefficients are the outcome of supply and demand influences working within a changing framework of policy, tastes and technical possibilities. In the field of education, as in any other, these coefficients may change through time for a number of reasons. For example: an effort may be made to reduce the size of classes, in which case the ratio of teachers to pupils will tend to rise; changes may take place in educational technology by the introduction, let us say, of teaching machines, in which case the ratio of teachers to pupils will tend to fall; changes in educational curricula, such as a move from specialism to generalism in upper secondary schools, are likely to affect the quantity and composition of all types of input; a change in the relative importance of boarders to day-pupils in some part of the system will change the range of costs that fall within the educational accounts. And so on.

Some of these changes can be controlled, others can only be foreseen In either case we should try to go further than is usually done towards measuring their economic and social implications. Thus if our object were to change some of the input-output coefficients, for example to increase the ratio of teachers to pupils, we could calculate the number of teachers required in, say, 1980, given the estimated number of pupils and the desired teacher-pupil ratio. This we could do for all types of educational institution, and we could then enquire into the possibility of increasing the stock of teachers up to the desired level in the time available, either by expanding teacher training or by paying teachers better or by some other means. These problems and some suggestions for their solution are discussed more fully in Chapter XI.

*Part Two*

# ANALYSIS

## MODEL-BUILDING BASED ON THE DEMOGRAPHIC ACCOUNTS

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# IX

# INPUT-OUTPUT MODELS OF DEMOGRAPHIC FLOWS

## 1. The uses of the models

This chapter is concerned with means of analysing the socio-demographic system developed in the preceding chapters. The models described belong to the finite-difference input-output type. Despite the fact that this paper is mainly devoted to questions of compilation and presentation, there are a number of reasons for including in it a discussion of model-building too.

First, these models provide some insight into the working of the system. It is a dynamic input-output system, where the output of one period flows into the next. Movement can take place either forwards or backwards in time: forward movement can be modelled with the help of outflow coefficients (transition proportions); backward movement calls instead for inflow coefficients (admission proportions).

Second, the models show the relationship between quantities (human flows) and prices (educational and other costs per unit). Just as we can trace population movements forwards or backwards, so we shall find that there are two kinds of price model: there is a forward price model, related to the forward quantity model, which enables us to calculate the future educational (or other) costs likely to be associated with the average individual at present in any state of the system; and there is a backward price model, related to the backward quantity model, which enables us to calculate past educational (or other) costs associated with the average individual at present in any state of the system.

Third, the models make possible a large number of detailed analyses. These may relate either to the structure of the system or to implications of present or expected changes.

Fourth, the models provide an analogy with the input-output analysis of economic systems which contributes to our understanding of both systems; and they provide a direct link with it, in that an economic input-output matrix for the educational system is simply a part of the input-output matrix for the whole productive system.

Finally, the models may be useful in settling detailed taxonomic questions. This has certainly been the experience in the economic case.

## 2. Framework of the models

A socio-demographic system of accounts such as that summarized numerically in Table III.10 can be represented symbolically as follows.

TABLE IX.1. A SYMBOLIC VERSION OF TABLE III.10

| | | Outside world | Our country | | Total |
|---|---|---|---|---|---|
| | | | This year | Next year | |
| Outside world | | | $b'$ | $\Lambda b'$ | |
| Our country | Last year | $\Lambda^{-1}d$ | $\Lambda^{-1}S$ | | $\Lambda^{-1}p$ |
| | This year | $d$ | | $S$ | $p$ |
| Total | | | $p'$ | $\Lambda p'$ | |

The symbols in this table have the following meaning.

$p$ the vector of population flows ; the typical element, $p_r$, say, of $p$ represents the total number of people who flow through state $r$ in the course of this year ;

$b$ the vector of births and net immigrations ; the typical element, $b_r$ say, of $b$ represents the number of people who flow into state $r$ from the outside world in the course of this year ;

$d$ the vector of deaths ; the typical element, $d_r$ say, of $d$ represents the number of people who flow out of state $r$ into the outside world in the course of this year ;

$S$ the matrix of survivors ; the typical element, $S_{rs}$ say, of $S$ represents the number of people who leave this year in state $r$ and flow next year into state $S$ ;

$\Lambda$ the lag operator which shifts in time the variable to which it is applied ; by convention $\Lambda^{\theta}S(\tau) = S(\tau+\theta)$, so that if $S$ denotes this year's survivors, $\Lambda^{-1}S$ denotes last year's survivors and $\Lambda S$ denotes next year's survivors ;
the prime superscript ($'$) indicates transposition ; that is, $p'$ a row vector formed from the elements of the column vector $p$ ; and $S'$ is a matrix formed by interchanging the rows and columns of $S$.

All the models described in the next two sections are based on this framework.

## 3. MODELS BASED ON OUTFLOW COEFFICIENTS (TRANSITION PROPORTIONS)

Input-output models of economic flows are usually based on input coefficients (cost proportions), although it is also possible to base economic models on output coefficients (sales proportions). Conversely, input-output models of demographic flows are usually based on outflow coefficients (transition proportions) although, as we shall see in the following section, it is possible to base demographic models on inflow coefficients (admission proportions). In this section, the models based on outflow coefficients are developed step by step.

### a) *The quantity model with fixed coefficients*

From the row for this year in Table IX.1 we can see that the flows of population out of this year consist of this year's deaths plus the survivors into next year. That is

$$p = Si+d \qquad \text{(IX.1)}$$

where $i$ denotes the unit column vector, so that $Si$ denotes the row sums of $S$. Thus the vector of survivors into next year is defined as $p-d = Si$.

From the column for this year in Table IX.1 we can see that the flows of population into this year consist of the survivors from last year plus this year's births and net immigrations. That is:

$$p' = i'\Lambda^{-1}S+b' \tag{IX.2}$$

or, on transposing the variables:

$$p = \Lambda^{-1}S'i+b \tag{IX.3}$$

Thus the vector of survivors from last year is defined as $p-b = \Lambda^{-1}S'i$.

A matrix of transition coefficients, $C$ say, can be formed by dividing the elements in each row of $\Lambda^{-1}S$ by the corresponding element of $\Lambda^{-1}p$. Thus:

$$C = \Lambda^{-1}\hat{p}^{-1}\Lambda^{-1}S \tag{IX.4}$$

where $\Lambda^{-1}\hat{p}$ denotes a diagonal matrix formed from the vector $\Lambda^{-1}p$, and $\Lambda^{-1}\hat{p}^{-1}$ is the inverse diagonal matrix, that is to say a matrix with the reciprocals of the elements of $\Lambda^{-1}p$ in its leading diagonal. The matrix $C$ shows the proportions in which the survivors from last year distribute themselves among the various states open to them this year.

By transposing (IX.4) and post-multiplying by $\Lambda^{-1}\hat{p}$, we can write:

$$\Lambda^{-1}S' = C'\Lambda^{-1}\hat{p} \tag{IX.5}$$

and by substituting for $\Lambda^{-1}S'$ from (IX.5) into (IX.3), we can write:

$$p = C'\Lambda^{-1}p+b \tag{IX.6}$$

Equation (IX.6) expresses a set of linear equations connecting this year's population flows, $p$, with last year's population flows, $\Lambda^{-1}p$, and this year's births and net immigrations, $b$. On the assumption that $C$ remains constant, this set of equations can be used to make forward projections. Before we consider this question let us take a look at the special case of a population in stationary equilibrium.

b) *The model applied to a population in stationary equilibrium*

A population is in stationary equilibrium if the number of individuals in each state, the elements of the vector $p$, remains constant, that is if $\Lambda^{\theta}p = p$ for all values of $\theta$. In this case (IX.6) can be simplified to:

$$\begin{aligned} p &= C'p+b \\ &= (I-C')^{-1}b \end{aligned} \tag{IX.7}$$

Equation (IX.7) has the same form as the open input-output model in economics. In that case a matrix multiplier, usually written as $(I-A)^{-1}$, transforms levels of final demand into levels of output. In the present case the matrix multiplier $(I-C')^{-1}$ transforms the numbers of people flowing from the outside world into the different states, the elements of $b$ (which repeat themselves in each period), into the total numbers flowing into those states. The matrix $(I-C')^{-1}$ is a lower triangular matrix with ones in the leading diagonal. The numbers below the diagonal in each column of this matrix represent the proportions in which the people found this year in a given state are likely to distribute themselves among the various states open to them in future years. This calculation is made on the assumption that the structure of the system, that is the $C$-matrix, remains unchanged over time.

c) *Forward projections*

Equation (IX.6) can be written in a variety of ways. For example:

$$\begin{aligned} p &= C'\Lambda^{-1}p+b \\ &= (I-C'\Lambda^{-1})^{-1}b \\ &= (I+C'\Lambda^{-1}+C'^{2}\Lambda^{-2}+\ldots)b \\ &= \sum_{\theta=0}^{\infty} C'^{\theta}\Lambda^{-\theta}b \end{aligned} \qquad \text{(IX.8)}$$

The second row of (IX.8) contains a dynamic matrix multiplier which involves the operator $\Lambda$ as well as the coefficient matrix $C$. The third row expands this matrix multiplier in powers of $C'\Lambda^{-1}$; and the fourth row expresses the third in a summation notation, on the understanding that $C'^{0} = I$ and $\Lambda^{0} = 1$.

By applying $\Lambda$ to the first row of (IX.8) we obtain:

$$\Lambda p = C'p+\Lambda b \qquad \text{(IX.9)}$$

and, continuing in this way, we can see that:

$$\Lambda^{\tau}p = \sum_{\theta=0}^{\tau-1} C'^{\theta}\Lambda^{\tau-\theta}b+C'^{\tau}p. \qquad \text{(IX.10)}$$

Equation (IX.10) shows $\Lambda^{\tau}p$, the state vector of the population $\tau$ periods hence, as the sum of $p$ transformed $\tau$ times by the matrix $C'$, and of all future $b's$ up to and including period $\tau$, each transformed the appropriate number of times, $\tau-1$, $\tau-2$, ..., 0 respectively.

d) *Partitions*

The set of equations represented by (IX.6) can be written in partitioned form. For instance, suppose the whole set of states is partitioned into two subsets: all states from birth to the end of compulsory education together with all non-compulsory learning states, to be denoted by the suffix 1; and all other states, to be denoted by the suffix 2. If it is impossible for anyone who has left subset 1 to return to it, $C$ can be written in the partitioned form:

$$C = \left[\begin{array}{c:c} C_{11} & C_{12} \\ \hdashline 0 & C_{22} \end{array}\right] \qquad \text{(IX.11)}$$

whence:

$$(I-C') = \left[\begin{array}{c:c} I-C'_{11} & 0 \\ \hdashline -C'_{12} & I-C'_{22} \end{array}\right] \qquad \text{(IX.12)}$$

and:

$$(I-C')^{-1} = \left[\begin{array}{c:c} (I-C'_{11})^{-1} & 0 \\ \hdashline (I-C'_{22})^{-1}C'_{12}(I-C'_{11})^{-1} & (I-C'_{22})^{-1} \end{array}\right] \qquad \text{(IX.13)}$$

In the general case, however, it is possible for those who have left the educational system to return to it. Where this is so:

$$(I-C') = \left[\begin{array}{c:c} I-C'_{11} & -C'_{21} \\ \hdashline -C'_{12} & I-C'_{22} \end{array}\right] \tag{IX.14}$$

and

$$(I-C')^{-1} = \left[\begin{array}{c:c} W & X \\ \hdashline Y & Z \end{array}\right] \tag{IX.15}$$

where:

$$W = (I-C'_{11})^{-1}[I+C'_{21}ZC'_{12}(I-C'_{11})^{-1}] \tag{IX.16}$$
$$X = (I-C'_{11})^{-1}C'_{21}Z \tag{IX.17}$$
$$Y = ZC'_{12}(I-C'_{11})^{-1} \tag{IX.18}$$

and

$$Z = [(I-C'_{22})-C'_{12}(I-C'_{11})^{-1}C'_{21}] \tag{IX.19}$$

This development makes it possible to relate the present analysis to Thonstad's analysis of the educational system of Norway, which is expressed in the language of absorbing Markov chains. In Thonstad's analysis the states in category 2 above are interpreted as completed educations, that is to say states in which the individuals remain for good once they have left the educational system. No distinctions of age are made and their place in the educational states (category 1 above) is taken by grades. In terms of (IX.11), $C_{22} = I$ so that $(I-C')$ is singular; but information on educational careers can be expressed, in Thonstad's notation, in terms of two matrices, $Q$ and $R$, where $Q$ is equivalent to $(I-C_{11})^{-1}$ and $R$ is equivalent to $C_{12}$.

e) *Changing coefficients*

In practice it is not reasonable to suppose that the $C$-matrix remains constant over time, any more than it is reasonable to make this assumption about the $A$-matrix in economic input-output analysis. If $C'$ changes from period to period, we can express its value $\beta$ periods from now as $\Lambda^{\beta}C'$. Thus the series $C'$, $C'^2$, $C'^3$, ..., will be replaced by the series $C'$, $\Lambda C' \cdot C'$, $\Lambda^2 C' \cdot \Lambda C' \cdot C'$, .... With this notation (IX.10) becomes

$$\Lambda^{\tau}p = \Lambda^{\tau}b + \sum_{\theta=1}^{\tau-1}\left[\prod_{\beta=\tau-1}^{\tau-\theta}(\Lambda^{\beta}C')\right]\Lambda^{\tau-\theta}b + \prod_{\theta=0}^{\tau-1}(\Lambda^{\theta}C')p \tag{IX.20}$$

where $\Pi$ denotes the operation of forming a product. The middle term in (IX.20), the multiplier of $\Lambda^{\tau-\theta}b$, is:

$$\prod_{\beta=\tau-1}^{\tau-\theta}\Lambda^{\beta}C' = \Lambda^{\tau-1}C' \cdot \Lambda^{\tau-2}C' \ldots \Lambda^{\tau-\theta}C' \tag{IX.21}$$

in place of $C'^{\theta}$ in (IX.10).

With changing coefficients, the expression comparable to $(I-C')^{-1}$ can be calculated as follows. Just as

$$(I-C')^{-1} = I + \sum_{\theta=1}^{\infty} C'^{\theta} \tag{IX.22}$$

so we can write:

$$(I-\bar{C}')^{-1} = I+\sum_{\theta=1}^{\infty} \bar{C}'^{\theta} \tag{IX.23}$$

where:

$$\bar{C}'^{\theta} = \prod_{\beta=0}^{\theta-1} \Lambda^{\beta} C' \,. \tag{IX.24}$$

Unlike $(I-C')^{-1}$, which remains constant through time, $(I-\bar{C}')^{-1}$ gradually changes; at any given time it expresses our best estimates of the movements from any given state to future states. Thus, using $(I-C')^{-1}$ based on data for 1965, the babies born in that year are assumed to have the same expectations at age 18, that is to say in 1983, as had the 18-year-olds of 1965; whereas using $(I-\bar{C}')^{-1}$, these expectations are updated to 1983, bearing in mind the changes at intermediate ages that now seem likely to take place between the two dates.

f) *Multiple changes of state in a period*

In $\Lambda^{-1}S$, the rows show the state in which people were last year and the columns show the state in which they are this year. In $S$, the rows show the state in which people are this year and the columns show the state in which they will be next year. In other words, everybody is assumed to make only one change of state per year. However, some people may change state more than once in a year, and this can be allowed for by introducing another matrix, $S^*$ say, at the intersection of the row and column for this year in Table IX.1. With this change we must introduce a new coefficient matrix, $C^{*\prime}$ say, defined as:

$$C^{*\prime} = S^{*\prime}\,\hat{p}^{*-1} \tag{IX.25}$$

where $p^*$ differs from $p$ in that individuals are classified by their state after their last, rather than after their first, change in this year. The basic equation (IX.8), now becomes:

$$\begin{aligned} p^* &= \Lambda^{-1}S'i+S^{*\prime}i+b \\ &= C^{**\prime}\Lambda^{-1}p^*+C^{*\prime}p^*+b \\ &= (I-C^{*\prime})^{-1}(C^{**\prime}\Lambda^{-1}p^*+b) \\ &= (I-C^{*\prime}-C^{**\prime}\Lambda^{-1})^{-1}b \end{aligned} \tag{IX.26}$$

where $C^{**\prime}$ differs from $C'$ in that it is defined as:

$$C^{**\prime} = \Lambda^{-1}S'\Lambda^{-1}\hat{p}^{*-1} \tag{IX.27}$$

rather than as the transpose of (IX.4). In this case, corresponding to (IX.10), we have:

$$\begin{aligned} \Lambda^{\tau}p^* = \sum_{\theta=0}^{\tau-1} &\{[(I-C^{*\prime})^{-1}C^{**\prime}]^{\theta}(I-C^{*\prime})^{-1}\Lambda^{\tau-\theta}b\} \\ &+[(I-C^{*\prime})^{-1}C^{**\prime}]^{\tau}p^* \,. \end{aligned} \tag{IX.28}$$

g) *A model for survivors*

In general, the row sums of $C$ are less than one because in each period there are some deaths in each state. For some purposes it is convenient to ignore deaths and restrict the analysis to survivors. To this end let us

define a matrix, $D'$, as:

$$\begin{aligned} D' &= \Lambda^{-1} S' \Lambda^{-1} (\hat{p} - \hat{d})^{-1} \\ &= S' (\hat{p} - \hat{d})^{-1} \end{aligned} \tag{IX.29}$$

on the assumption that the elements of $D$, like those of $C$, are fixed through time. On rearranging (IX.29) and post-multiplying by $\hat{p}^{-1}$, we obtain:

$$\begin{aligned} D'(I - \hat{d}\hat{p}^{-1}) &= S'\hat{p}^{-1} \\ &= C' \end{aligned} \tag{IX.30}$$

whence:

$$(I - D')^{-1} = [I - C'(I - \hat{d}\hat{p}^{-1})^{-1}]^{-1}. \tag{IX.31}$$

h) *The application to vintages*

Let us denote by $v_\theta$ the state vector, going from birth to death, of the vintage born in year $\theta$: each element of $v_\theta$ shows the number of people born in year $\theta$ found in a particular activity at a particular age. And let us denote by $b^*$ a vector formed by selecting from a succession of $b$-vectors all the people born in year $\theta$, starting with those aged 0 in year $\theta$. Then:

$$v_\theta = (I - C')^{-1} b^*. \tag{IX.32}$$

If we wish to restrict the analysis to individuals born in our country, we can represent this formally by writing:

$$v_\theta = (I - C')^{-1} J b^* \tag{IX.33}$$

where $J$ is a square matrix of order equal to the number of elements in $b^*$ with a single non-zero element in its top left-hand corner. This element will be one or a little less than one, since it is reasonable to suppose that there are relatively few 0-year-old immigrants. A complementary analysis for immigrants of that vintage is obtained by substituting $(I - J)$ for $J$ in (IX.33).

i) *The equation of costs*

In economic input-output analysis we are accustomed to the idea that there is a price equation connecting total costs per unit of output with the cost of primary inputs per unit of output, which is symmetrical with the quantity equation connecting total output with final demand. A similar situation holds in the present case. The quantity equation is given by (IX.6); and the price equation can be written in the form:

$$\begin{aligned} k &= C\Lambda k + m \\ &= (I - C\Lambda)^{-1} m. \end{aligned} \tag{IX.34}$$

In (IX.34), $m$ denotes a vector of present educational unit costs, that is, a vector each of whose elements shows the cost of all the economic inputs required this year to educate an individual now in a given state; and $k$ denotes a vector of accumulated future educational unit costs, that is a vector each of whose elements shows the cost of all the economic inputs required, on average, to take an individual now in a given state from (and including) this year to the end of his education.

On the assumption that the population is in stationary equilibrium and that unit costs in each activity are constant, then $\Lambda^\theta = 1$ for $\theta = 1, 2, \ldots$

in both (IX.6) and (IX.34). Consequently we can write:

$$\begin{aligned} m'p &= k'(I-C')(I-C')^{-1}b \\ &= k'b \end{aligned} \qquad \text{(IX.35)}$$

that is to say, this year's total expenditure on education, $m'p$, is equal to the total future cost of educating this year's births and net immigrants. Equation (IX.35) is analogous to the fundamental theorem of national economic accounting which states that national income is equal to national expenditure.

j) *The economic input structure of education*

The detailed cost structures of educational activities can be set out in a coefficient matrix, $M$ say, with types of input in the rows and states in the columns: the typical element, $m_{js}$ say, of $M$ shows the amount of input $j$ required this year to educate an individual in state $s$, The matrix $M$ has the same number of columns as $C$, but in it the columns for non-educational states are empty; and

$$m = M'i. \qquad \text{(IX.36)}$$

By multiplying $M$ by the total number of people found in each state we obtain a flow matrix, $N$ say, whose typical element, $n_{js}$ say, shows the amount of input $j$ required to educate all the people in state $s$. Thus:

$$N = M\hat{p} \qquad \text{(IX.37)}$$

and, on the assumption that $M$ remains constant over time,

$$\Lambda^{\tau}N = M\Lambda^{\tau}\hat{p}. \qquad \text{(IX.38)}$$

Further, if $n$ denotes a vector of total costs such that $n = Ni$, then:

$$n = Mp \qquad \text{(IX.39)}$$

whence:

$$\begin{aligned} \Lambda^{\tau}n &= M\Lambda^{\tau}p \\ &= M\left[\sum_{\theta=0}^{\tau-1} C'^{\theta}\Lambda^{\tau-\theta}b + C'^{\tau}p\right] \end{aligned} \qquad \text{(IX.40)}$$

from (IX.10).

It may be necessary to elaborate in various ways this scheme for projecting educational costs. First, the $M$-matrix is as likely to change as is the $C$-matrix; and, second, it may be possible to divide $M$ into fixed and variable components, thus making costs a general linear function of the number of pupils or students.

## 4. Models based on inflow coefficients (admission proportions)

The models described in the preceding section are all forward-looking. The quantity equation, (IX.6) or (IX.8), enables us to make forward projections of human flows; and the cost equation, (IX.34) relates the total cost of completing an education to the cost that is now being incurred.

In giving expression to the idea of the cost of education, it would be useful to be able to calculate also the past educational costs associated with any particular state. This cannot be done with a model based on outflow coefficients. It can be done, however, with a model based on inflow coefficients, and it is to this type of model that we shall now turn.

The forward-looking model derived its coefficients from the rows of $\Lambda^{-1}S$. The backward-looking model derives its coefficients from the columns

of $S$. These inflow coefficients can be called admission proportions, since they indicate the proportions in which the admissions into a given state are drawn from other states.

a) *The quantity model with fixed coefficients*

If we go back to Table IX.1, the row equation for this year takes the form:

$$p = S\hat{p}+d \tag{IX.41}$$

as in (IX.1) above. Let us define a coefficient matrix, $G$ say, based on the columns of $S$, as

$$G = S\Lambda\hat{p}^{-1} \tag{IX.42}$$

If we substitute for $S$ from (IX.42) into (IX.41) we obtain:

$$p = G\Lambda p+d \tag{IX.43}$$

which can be compared with (IX.6).

b) *The model applied to a population in stationary equilibrium*

If $\Lambda^{\theta}p = p$ for all values of $\theta$, then (IX.43) simplifies to

$$\begin{aligned} p &= Gp+d \\ &= (I-G)^{-1}d \end{aligned} \tag{IX.44}$$

Equation (IX.44) has the same form as (IX.7) but it connects $p$ with $d$ rather than with $b$. In this case the matrix multiplier, $(I-G)^{-1}$, is upper triangular with ones in the leading diagonal and only zeros below it.

c) *Backward projections*

Like (IX.6), equation (IX.43) can be written in a number of ways; for example:

$$\begin{aligned} p &= G\Lambda p+d \\ &= (I-G\Lambda)^{-1}d \\ &= (I+G\Lambda+G^2\Lambda^2+\ldots)\,d \\ &= \sum_{\theta=0}^{\infty} G^{\theta}\Lambda^{\theta}d \end{aligned} \tag{IX.45}$$

By applying $\Lambda^{-1}$ successively to the first row of (IX.45), we can see that:

$$\Lambda^{-\tau}p = \sum_{\theta=0}^{\tau-1} G^{\theta}\Lambda^{\theta-\tau}d+G^{\tau}p \tag{IX.46}$$

which can be compared with (IX.10).

d) *The equation of costs*

In this case the cost equation can be expressed in the form:

$$\begin{aligned} l &= G'\Lambda^{-1}\,l+m \\ &= (I-G'\Lambda^{-1})^{-1}m. \end{aligned} \tag{IX.47}$$

In (IX.47), $m$ has the same meaning as in (IX.34) and $l$ is a vector each of whose elements represents the total educational unit costs incurred in reaching a given state of the system. Again, as in the economie example, the coefficient

TABLE IX.2. THE SIMPLEST CASE: A POPULATION IN STATIC

*A.* FORWARD-LOOKING ANALYSIS BASED ON OUTFLOW COEFFICIENTS (transition proportions)

1. *Quantity equation*

$$\underset{p}{\begin{bmatrix}1{,}000\\900\\225\\675\\900\\900\end{bmatrix}} = \underset{C'}{\begin{bmatrix}0&0&0&0&0&0\\0.9&0&0&0&0&0\\0&0.25&0&0&0&0\\0&0.75&0&0&0&0\\0&0&1&1&0&0\\0&0&0&0&1&0\end{bmatrix}} \underset{p}{\begin{bmatrix}1{,}000\\900\\225\\675\\900\\900\end{bmatrix}} + \underset{b}{\begin{bmatrix}1{,}000\\0\\0\\0\\0\\0\end{bmatrix}}$$

$$= \begin{bmatrix}1&0&0&0&0&0\\0.9&1&0&0&0&0\\0.225&0.25&1&0&0&0\\0.675&0.75&0&1&0&0\\0.9&1&1&1&1&0\\0.9&1&1&1&1&1\end{bmatrix} \begin{bmatrix}1{,}000\\0\\0\\0\\0\\0\end{bmatrix}$$

$$= \underset{(I-C')^{-1}}{\quad} \underset{b}{\quad}$$

2. *Cost equation*

$$\underset{k}{\begin{bmatrix}1.575\\1.75\\1\\2\\1\\0\end{bmatrix}} = \underset{C}{\begin{bmatrix}0&0.9&0&0&0&0\\0&0&0.25&0.75&0&0\\0&0&0&0&1&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}} \underset{k}{\begin{bmatrix}1.575\\1.75\\1\\2\\1\\0\end{bmatrix}} + \underset{m}{\begin{bmatrix}0\\0\\0\\1\\1\\0\end{bmatrix}}$$

$$= \begin{bmatrix}1&0.9&0.225&0.675&0.9&0.9\\0&1&0.25&0.75&1&1\\0&0&1&0&1&1\\0&0&0&1&1&1\\0&0&0&0&1&1\\0&0&0&0&0&1\end{bmatrix} \begin{bmatrix}0\\0\\0\\1\\1\\0\end{bmatrix}$$

$$= \underset{(I-C)^{-1}}{\quad} \underset{m}{\quad}$$

3. *Balance equation*

$$\underset{m'}{[0\ 0\ 0\ 1\ 1\ 0]} \underset{p}{\begin{bmatrix}1{,}000\\900\\225\\675\\900\\900\end{bmatrix}} = \underset{k'}{[1.575\ \ 1.75\ \ 1\ \ 2\ \ 1\ \ 0]} \underset{b}{\begin{bmatrix}1.000\\0\\0\\0\\0\\0\end{bmatrix}}$$

$$= 1.575$$

matrix of the cost equation, $G'$ in this case, is the transpose of the coefficient matrix, $G$, of the quantity equation.

In the special case of a population in stationary equilibrium operating under conditions of constant costs:

$$\begin{aligned} m'p &= l'(I-G)(I-G)^{-1}d \\ &= l'd \end{aligned} \qquad \text{(IX.48)}$$

that is to say, this year's expenditure on education, $m'p$, is equal to the total

*B.* BACKWARD-LOOKING ANALYSIS BASED ON INFLOW COEFFICIENTS (admission proportions)

1. *Quantity equation*

$$\begin{matrix} p \\ \begin{bmatrix} 1{,}000 \\ 900 \\ 225 \\ 675 \\ 900 \\ 900 \end{bmatrix} \end{matrix} = \begin{matrix} G \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} p \\ \begin{bmatrix} 1{,}000 \\ 900 \\ 225 \\ 675 \\ 900 \\ 900 \end{bmatrix} \end{matrix} + \begin{matrix} d \\ \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 900 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 1 & 0.75 & 0.75 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 900 \end{bmatrix}$$

$$= \underset{(I-G)^{-1}}{\quad} \quad \underset{d}{\quad}$$

2. *Cost equation*

$$\begin{matrix} l \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1.75 \\ 1.75 \end{bmatrix} \end{matrix} = \begin{matrix} G' \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.75 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix} \begin{matrix} l \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1.75 \\ 1.75 \end{bmatrix} \end{matrix} + \begin{matrix} m \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0.25 & 0.75 & 1 & 0 \\ 1 & 1 & 0.25 & 0.75 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \underset{(I-G')^{-1}}{\quad} \quad \underset{m}{\quad}$$

3. *Balance equation*

$$\begin{matrix} m' \\ [0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0] \end{matrix} \begin{matrix} p \\ \begin{bmatrix} 1{,}000 \\ 900 \\ 225 \\ 675 \\ 900 \\ 900 \end{bmatrix} \end{matrix} = \begin{matrix} l' \\ [0 \quad 0 \quad 0 \quad 1 \quad 1.75 \quad 1.75] \end{matrix} \begin{matrix} d \\ \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 900 \end{bmatrix} \end{matrix}$$

$$= 1{,}575$$

past cost of educating everyone who has died in this year. This result can be compared with (IX.35).

## 5. THE INTERACTION OF THE FORWARD AND BACKWARD MODELS

On the assumption of a stationary population, we can see, by combining (IX.7) and (IX.44), that:

$$d = (I-G)\,(I-C')^{-1}\,b \qquad \text{(IX.49)}$$

namely, that the vector of deaths is a linear transform of the vector of births and net immigrations.

If we make the further assumption that unit costs remain constant over time, we can see, by combining (IX.34) and (IX.47), that:

$$k = (I-C)^{-1}(I-G')\,l \qquad \text{(IX.50)}$$

namely, that the vector of future unit costs is a linear transform of the vector of past unit costs.

These simple transformations do not hold in the general case of a changing population and changing costs. However, if the *C*-matrix changes over time, it is possible to calculate the *G*-matrix compatible with it in each period and *vice-versa*. In other words, given a sequence of *C*-matrices it is possible to work out the costs to be incurred by the average individual in reaching any particular state; and given a sequence of *G*-matrices it is possible to work out the future costs which will be incurred by the average individual who is now in any particular state. These calculations are based on the fundamental and restrictive assumption of first-order Markov processes: that transitions from one state to another are independent of the earlier transitions that led to the first state; or, in the alternative model, that admissions to one state from another are independent of the later admissions to which that first state will open the way.

It is usually assumed that the *C*-matrix is primary, in which case the *G*-matrix is derivative. According to this line of thought, it makes sense to project the *C*-matrix and thus derive future values of the *G*-matrix. It is questionable whether this view is altogether justified, since it seems to amount to a belief that the demand for places will somehow always triumph over the capacity for admissions. While it may be hoped that supply adapts itself to demand rather than the other way round, it seems desirable to consider the changes in *G* as well as in *C* if the models are to be used for forecasting.

## 6. An arithmetical illustration

By way of a summary, it may be helpful to give an arithmetical illustration simple enough to enable the main numerical relationships to be presented on a single page. This is done in Table IX.2.

The illustration relates to a population in stationary equilibrium with unchanging unit costs. There are five ages, 0, 1, ..., 4, and two activities, "not being at school" and "being at school". Each year 1,000 babies are born and 100 of them die within the year; after that no further deaths occur until age 4, when all survivors die. There is no migration at any age. School attendance takes place at ages 2 and 3: at age 2, three-quarters of the children attend school; at age 3, all of them attend. There are six states: (*i*) being 0 and not at school; (*ii*) being 1 and not at school; (*iii*) being 2 and not at school; (*iv*) being 2 and at school; (*v*) being 3 and at school; and (*vi*) being 4 and not at school. Hence the *C*-matrix and the *G*-matrix are both of order 6. Educational costs are measured in years, and so the elements of *m*, the vector of this year's unit costs, are 1 for states (*iv*) and (*v*) and 0 for all other states.

The left-hand side of Table IX.2 relates to the forward-looking analysis based on outflow coefficients (transition proportions) described in section 3 above. The quantity equation will be recognized as (IX.7), the cost equation as the static version of (IX.34), and the balance equation as (IX.35).

The quantity equation shows two ways of calculating this year's vector of population flows, $p$, with the help of the $C$-matrix: first, by adding the vector of this year's new entrants, $b$ (in this case containing only births since there are no immigrations) to the vector of last year's population flows (also $p$ since the population is in stationary equilibrium) subjected to a single set of forward transitions by pre-multiplication by $C'$, the transpose of $C$; and, second, by pre-multiplying the same vector of new entrants by the matrix multiplier, $(I-C')^{-1}$. Each column of $(I-C')^{-1}$ shows what probabilities an individual now in a given state has of reaching any of the other states. Thus, from the first column, a new-born baby has a 0.9 probability of reaching age 1 (the second state), a $0.9 \times 0.25 = 0.225$ probability of reaching age 2 and not going to school (the third state), and so on; the second column sets out the probabilities for a child aged 1; and the remaining columns do the same for those who have reached higher states.

The cost equation shows the present and future unit costs associated with each state, the elements of $k$: first, as the sum of this year's unit costs, the elements of $m$, and of all the unit costs to be incurred in the future, the elements of $Ck$; and, second, as the vector of this year's unit costs pre-multiplied by the matrix multiplier, $(I-C)^{-1}$.

Finally, the balance equation shows that the total future cost of educating those born this year, $k'b$, is equal to this year's total expenditure on education, $m'p$.

The right-hand side of the table relates to the backward-looking analysis based on inflow coefficients (admission proportions) described in section 4 above. The quantity equation will be recognized as (IX.44), the cost equation as the static version of (IX.47), and the balance equation as (IX.48).

In this case, the quantity equation shows two ways of calculating the vector of this year's population flows, $p$, with the help of the $G$-matrix: first, by adding the vector of this year's deaths, $d$, to the vector of next year's population (again $p$) subjected to a single set of backward transitions by pre-multiplication by $G$; and, second, by pre-multiplying the same vector of deaths by the matrix multiplier, $(I-G)^{-1}$. Each column of $(I-G)^{-1}$ shows what probabilities there are that an individual now in a given state has been through any of the other states. Thus, from the sixth column, an individual now aged 4 must have been at school at age 3; similarly, he must have been not at school at ages 0 and 1; but at age 2 there is a 0.75 probability that he was at school and a 0.25 probability that he was not.

The cost equation shows the past and present unit costs associated with each state, the elements of $l$: first, as the sum of this year's unit costs, the elements of $m$, and of all the unit costs incurred in the past, the elements of $G'l$; and, second, as the vector of this year's unit costs pre-multiplied by the matrix multiplier, $(I-G')^{-1}$.

Finally, the balance equation shows that the total past cost of educating those dying this year, $l'd$, is equal to this year's total expenditure on education, $m'p$.

# X

# THE MODELS APPLIED TO LEARNING ACTIVITIES: SOME NUMERICAL EXAMPLES

## 1. The Scope of the Examples

The purpose of this chapter is to illustrate with figures taken from existing statistics how the models described in the preceding chapter can be applied to the real world. The examples given relate only to the demographic flows through the educational system. The economic flows cannot be illustrated because, as was explained in chapter VIII, with the data available at present the economic accounts cannot be made conformable with the demographic accounts.

The basic data to which the models have been applied cover the younger age-groups in the male population of England and Wales in 1964-65. Both the forward-looking and the backward-looking quantity models are illustrated. The core of the former is the *C*-matrix of transition proportions, and this is given after the basic data and is followed by the matrix multiplier derived from it; this in turn is followed by two subsidiary tables exemplifying some of its possible uses. The core of the backward-looking model is the *G*-matrix of admission proportions; this is not shown here, but a subsidiary table derived from it is given.

All the subsidiary tables are based on the assumption of fixed coefficients. That is to say, they show what the educational development of any vintage would be like if the transition (or admission) proportions were set in the 1964-65 pattern. Their interest lies partly in their function as illustrations and partly in the fact that they bring out clearly the present distribution of education among the younger members of the population. They should not be taken as projections into the future (or into the past), since they do not allow for changes in the coefficients. These changes will be discussed in the next chapter.

## 2. The Matrix of Survivors

The numerical example about to be described can best be introduced by referring back to Table III.11. The upper left-hand quadrant of that table showed the male population of England and Wales, classified by age and activity, flowing out of 1964 and into 1965. If we want to see in greater detail the flows through the educational system, we can subdivide the younger age-groups, the under-twenties, by year of age and disaggregate the activity "at school" into a number of learning activities. The results of

this operation are shown in Table X.1*. In terms of the symbolism used in Table IX.1 of the preceding chapter, this table corresponds to the matrix $\Lambda^{-1}S$ supplemented by the vectors $\Lambda^{-1}d$, $\Lambda^{-1}p$, $b'$ and $p'$.

In Table X.1, the educational boundary is drawn to exclude both part-time and informal education. Each year of age, from 0 to 19, is shown separately. Within ages, the classification is by type of educational institution, with only two grades distinguished in the upper range of secondary education. No attempt is made at recording subject streams or qualifications obtained. The activities distinguished are as follows:

0 Not in full-time formal education.
1 Nursery and primary schools.
2a Secondary modern schools: not A-level work.
2b Secondary modern schools: A-level work.
3a Grammar schools: not A-level work.
3b Grammar schools: A-level work.
4a Comprehensive schools: not A-level work.
4b Comprehensive schools: A-level work.
5a Other normal schools: not A-level work.
5b Other normal schools: A-level work.
6 Special schools (for the handicapped).
7 Further education n.e.s.
8 Colleges of education (teacher training colleges).
9 Universities (including colleges of advanced technology).

The twenty ages and fourteen activities combine to define 114 states, which are numbered in the rows and columns of the table: for example, state 3 is "being aged 2 and not in full-time formal education"; state 4 is "being aged 2 and at nursery school"; state 5 is "being aged 2 and at special school"; and so on. The first row and column, which relate to the outside world, are excluded from this numbering; as in Table III.11, the row contains the births and net immigrations of 1965, $b'$, and the column contains the deaths of 1964, $\Lambda^{-1}d$. The final row and column simply contain the totals, $p'$ and $\Lambda^{-1}p$ respectively.

In the body of the table, bounded by the outside world and by the totals, are the survivors from 1964 into 1965, $\Lambda^{-1}S$, classified in the rows by the state which they occupied at the end of 1964 and in the columns by the state to which they moved during 1965. For example, if we look at row 62 we can see what happened in 1965 to the 72,700 grammar-school boys who were aged 15 in 1964 and who were not doing A-level work at the end of that year: 13,200 left the system of full-time formal education; 58,000 (of whom 27,800 moved on to A-level work) remained at school; and 1,500 transferred to an establishment of further education. From row 63 we can see that there were 7,500 grammar-school boys of the same age already engaged in A-level work in 1964 and that, in 1965, 500 of them left the system of full-time formal education and the remainder continued at school.

If we combine the information in these two rows, we find that of the 80,200 grammar-school boys who became 16 in 1965, as many as 65,000, or 81 per cent, remained at school; 1,500, or 2 per cent, entered an establishment of further education; and 13,700, or 17 per cent, left the

* Tables X.1 to X.3 are under cover flap.

system of full-time formal education. We have already met these percentages in section 3(a) of Chapter V.

Table X.1 is as far as the basic data can be taken at present. Although in it the educational boundary is tightly drawn and no distinction is made either of subjects studied or of qualifications obtained, it is sufficient to illustrate how the accounting system proposed in Chapter III can be expanded to give a detailed treatment of any activities we may be interested in.

## 3. The Matrix of Transition Proportions and the Matrix Multiplier

As shown in equation (IX.4), the $C$-matrix of transition proportions is obtained by dividing the elements in each row of $\Lambda^{-1}S$ by the corresponding element of $\Lambda^{-1}p$. We have just seen a numerical version of $\Lambda^{-1}S$ and $\Lambda^{-1}p$ in Table X.1. Accordingly, if the elements in the rows of Table X.1 (including the first element in each row, relating to the deaths of 1964) are divided by their row totals, the result is a set of coefficients which show in what proportions the boys found in the various states in 1964 either die in that year or redistribute themselves among the various states open to them in 1965. These coefficients, with the exclusion of the coefficients for deaths, constitute the $C$-matrix for 1964-65. The consequence of excluding the death coefficients is, of necessity, that many of the rows of $C$ add up to less than one.

The transpose of $C$, that is the matrix $C'$, is set out in Table X.2. The form $C'$ has been preferred for our numerical example because it is that used in the quantity models described in the preceding chapter.

By subtracting $C'$ from the unit matrix, $I$, we obtain $I-C'$; and by inverting this matrix we obtain the matrix multiplier $(I-C')^{-1}$ which appears in equation (IX.7). This matrix multiplier is set out in Table X.3 following Table X.2. As can be seen, each element in the leading diagonal of Table X.3 is 1.000. The numbers below the diagonal in each column of the table show what probabilities a boy now in a particular state (indicated by the position of the diagonal element) would have of reaching any of the subsequent states if, as he progressed through life, the structure of the system, that is the $C$-matrix, remained unchanged.

It was said in Chapter II that most users of complex systems have little interest in the systems as such. Table X.3 helps to explain this remark: on and below the leading diagonal of this table there are 6,555 elements, and although some of them are empty, the majority are full. This is too much detail for most practical purposes. What is needed are extracts and summaries. Let us now look at some of these.

## 4. The Progression to Age 19

It is a simple matter to extract from the matrix multiplier a table which shows the educational progression from birth to the age of 19. All that need be done is rearrange by activity and age the first column of Table X.3. The result is set out in Table X.4. For ease of reference, all the figures have been multiplied by 10,000.

Table X.4 shows, expressed as proportions of 10,000, the educational distribution of the male population under 20 in 1965. From it we can see

TABLE X.4. EDUCATIONAL PROGRESSION FROM
BASED ON THE TRANSITION PROPORTIONS FO

| Activity \ Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0. Not in full-time formal education | 10,000 | 9,803 | 9,741 | 9,462 | 7,023 | 212 | 88 |
| 1. Nursery and primary | | | 25 | 290 | 2,715 | 9,506 | 9,610 |
| 2. Secondary modern: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 3. Grammar: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 4. Comprehensive: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 5. Other Normal: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 6. Special schools | | | 5 | 7 | 14 | 29 | 43 |
| 7. Further education n.e.s. | | | | | | | |
| 8. Colleges of education | | | | | | | |
| 9. Universities | | | | | | | |
| Total | 10,000 | 9,803 | 9,771 | 9,759 | 9,752 | 9,747 | 9,741 |

what would happen to ten thousand baby boys if the structure of the educational system remained for the first twenty years of their life as it was in 1965. As we can see from the final row, the original number is gradually diminished by death, from 10,000 to 9,675; in the body of the table we see the numbers to be found in each activity at each age; and in the final column we see the total number of boy-years spent in each activity.

As we saw in Table X.1, a small number of boys first enter school at the age of 2: Table X.4 shows this number to be twenty-five at normal schools and five at special schools for every ten thousand births, or 0.3 per cent of a vintage. After the age of 2 there is a gradual increase in the numbers entering both types of school. There is a large increase in the numbers entering normal schools at 4 and a still larger increase at 5, the age of compulsory school attendance. The switch-over to secondary schools is mainly concentrated at the ages of 11 and 12. A-level work starts at 15 and builds up to a maximum at 17, especially in grammar schools. The school-leaving age being 15, this is when the exodus from full-time formal education, particularly marked among the pupils of secondary modern schools, begins. At 15, too, the move into institutions of further education begins, although attendance at teacher training colleges (colleges of education) and at universities does not start until 17. By the age of 19 there are 9,675 survivors of the original 10,000, of whom 8,501 are no longer in full-time formal education. Of the 1,174 who remain in it, rather more than half, namely 654, are at university.

These results can be looked at from another point of view, suggested by the figures in the last column of the table. The total at the bottom of that column shows that the first twenty years of life are reduced,

IRTH THROUGH AGE 19 OF TEN THOUSAND BOYS
NGLAND AND WALES IN 1964-65

| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 67 | 35 | | | | | | 15 | 3,229 | 6,310 | 7,537 | 8,207 | 8,501 | 80,230 |
| 9,600 | 9,601 | 9,568 | 9,515 | 3,085 | 252 | 86 | 3 | | | | | | 63,916 |
| | | | | 3,666 | 5,239 | 5,131 | 5,037 | 2,419 | 449 | 46 | 3 | | 21,990 |
| | | | | | | | | | 21 | 23 | 12 | | 56 |
| | | 6 | 35 | 1,441 | 2,076 | 2,257 | 2,342 | 2,079 | 864 | 107 | 14 | 3 | 11,164 |
| | | | | | | | | 213 | 1,026 | 1,296 | 548 | 62 | 3,145 |
| | | | | 697 | 1,158 | 1,344 | 1,512 | 1,047 | 346 | 58 | 6 | | 6,168 |
| | | | | | | | | 3 | 129 | 202 | 108 | 24 | 466 |
| | | | 3 | 660 | 814 | 718 | 628 | 384 | 123 | 17 | 2 | 1 | 3,350 |
| | | | | | | | | 2 | 35 | 45 | 18 | 2 | 102 |
| 70 | 95 | 152 | 170 | 170 | 178 | 178 | 175 | 151 | 42 | 4 | | | 1,483 |
| | | | | | | | | 179 | 360 | 347 | 359 | 297 | 1,542 |
| | | | | | | | | | | 4 | 59 | 131 | 194 |
| | | | | | | | | | | 10 | 349 | 654 | 1,013 |
| 9,737 | 9,731 | 9,726 | 9,723 | 9,719 | 9,717 | 9,714 | 9,712 | 9,706 | 9,705 | 9,696 | 9,685 | 9,675 | 194,819 |

on average, to an effective 19.5 years, mainly as consequence of infant mortality. Of these 19.5 years, 11.5, on average, are spent in full-time formal education and 8 are spent out of the system of full-time education. Of the 11.5 years spent inside the system, 10.8, on average, are spent at school up to A-level work and only 0.7 are spent in A-level work and beyond. Of the 8 years spent outside the system, 4.6, on average, are spent before joining it and 3.4 are spent after leaving it.

This is the basic position. Let us now see how the prospects of boys are affected by the stage they have reached in the progression.

## 5. Time Spent in Different Activities

The figures in Table X.3 can be analysed in another way. Suppose we ask the question: how much time will be spent in different activities after a certain age by the boys who are now in a given state? Table X.5 overleaf answers this question for boys in eight initial states with respect to their average distribution of time during the seven years from the age of 13 through the age of 19, always on the assumption that the transition proportions of 1964-65 remain unchanged. Actually, since these calculations relate only to the survivors from each initial state, Table X.5 is based not on $C'$ but on $D'$ as defined in equation (IX.29). The numbers at the head of the columns of Table X.5 refer to the corresponding columns of $(I-D')^{-1}$.

The first column of Table X.5 shows the average time that a batch of new-born boys could expect to spend in the various activities open to them between the ages of 13 and 19. Of this span of seven years, the first two

TABLE X.5. AVERAGE DISTRIBUTION OF TIME TO BE SPENT IN DIFFERENT EDUCATIONAL ACTIVITIES FROM AGE 13 THROUGH AGE 19 BY BOYS IN EIGHT SELECTED INITIAL STATES

BASED ON THE TRANSITION PROPORTIONS FOR ENGLAND AND WALES IN 1964-65

*Years.*

| Activity \ State | Age 0 | Age 5: nursery and primary | Age 11: primary | Age 13: primary | Age 13: secondary modern | Age 13: grammar | Age 13: comprehensive | Age 13: other normal schools |
|---|---|---|---|---|---|---|---|---|
| | 1 | 13 | 33 | 47 | 48 | 49 | 50 | 51 |
| 0. Not in full-time formal education | 3.490 | 3.486 | 3.392 | 1.909 | 4.172 | 1.809 | 3.716 | 3.586 |
| 1. Nursery and primary | 0.009 | 0.009 | 0.029 | 1.032 | 0 | 0 | 0 | 0 |
| 2. Secondary modern: | | | | | | | | |
| *a*) not A-level | 1.349 | 1.366 | 1.277 | 0 | 2.552 | 0 | 0 | 0 |
| *b*) A-level | 0.006 | 0.006 | 0.006 | 0 | 0.011 | 0 | 0 | 0 |
| 3. Grammar: | | | | | | | | |
| *a*) not A-level | 0.789 | 0.799 | 0.924 | 2.235 | 0 | 3.309 | 0 | 0.006 |
| *b*) A-level | 0.324 | 0.328 | 0.384 | 1.260 | 0.019 | 1.302 | 0 | 0.003 |
| 4. Comprehensive: | | | | | | | | |
| *a*) not A-level | 0.444 | 0.449 | 0.508 | 0 | 0.044 | 0 | 2.869 | 0.323 |
| *b*) A-level | 0.048 | 0.049 | 0.051 | 0.004 | 0.014 | 0.004 | 0.238 | 0.089 |
| 5. Other normal: | | | | | | | | |
| *a*) not A-level | 0.193 | 0.195 | 0.115 | 0 | 0 | 0 | 0 | 2.607 |
| *b*) A-level | 0.010 | 0.010 | 0.006 | 0 | 0 | 0 | 0 | 0.139 |
| 6. Special schools | 0.057 | 0.017 | 0.009 | 0 | 0 | 0 | 0 | 0 |
| 7. Further education n.e.s. | 0.159 | 0.160 | 0.160 | 0.175 | 0.168 | 0.178 | 0.105 | 0.153 |
| 8. Colleges of education | 0.020 | 0.020 | 0.022 | 0.046 | 0.007 | 0.047 | 0.022 | 0.024 |
| 9. Universities | 0.104 | 0.106 | 0.119 | 0.336 | 0.016 | 0.347 | 0.052 | 0.066 |
| Total time in years | 7.000 | 7.000 | 7.000 | 7.000 | 7.000 | 7.000 | 7.000 | 7.000 |

NOTE. Components do not add up to totals because of rounding-off errors.

are still years of compulsory schooling, so that at most five out of the seven could be spent outside the system of full-time education. The entry in the first row and first column of the table is approximately 3.5, indicating that the average expectation at birth is for 1.5 years of voluntary full-time education between the end of compulsory school attendance and the age of 20.

If we compare the first two columns of the table we can see that the average expectations at birth are not much changed as we move on to 5-year-olds at normal schools. The prospects of these children are slightly better only because some of their mentally and physically handicapped contemporaries are at special schools.

A further improvement appears in the third column, relating to boys aged 11, the average age of transfer from primary to secondary school. At this age some children have already made the transfer, and those who have not yet done so have better prospects; the reason for this is that many of them are likely to be attending a higher type of primary school, the so-called preparatory school, which keeps children up to 13 and is specifically intended to prepare them for a grammar-school career. This point is made more evident in the fourth column: the boys who are still at primary school at 13 are almost all destined to enter grammar school.

The remaining four columns show the prospects of 13-year-olds at the four types of normal secondary school. The extreme contrast appears between the pupils of secondary modern schools and those of grammar schools: the former can expect, on average, less than a year of voluntary full-time education between the ages of 15 and 19, while the latter can expect more than three years of it over the same age-span. An equally marked difference can be seen in the average time spent by the two groups on A-level work and at institutions of higher education.

This is as far as we can go with the information given by Table X.3. More could be learnt if it were possible to classify children not only by age and educational institution but also by ability and family background. We should then almost certainly find groups for which the entry in the first row of Table X.5 approached the limiting values of 0 or 5; and we should gain a clearer picture of the social and psychological factors that affect the attitudes to education, and the opportunities for education, in different sections of the community.

## 6. The Educational Content of a State

Given the matrix $G$, defined in equation (IX.42) above, we can calculate the time that must have been spent in different activities, on average, by individuals now in a given state. Table X.6 sets out calculations of this kind for two terminal states: 19-year-old boys at university and 19-year-old boys who have left the system of full-time education. These results are obtained by re-tabulating and multiplying by 10,000 the figures in columns 114 and 106 respectively of the matrix multiplier $(I-G)^{-1}$.

Superficially, Table X.6 resembles Table X.4, but instead of examining the forward progression from birth to 19, it examines the past experience which has led to participation is specified activities at 19. In Table X.4, the column totals gradually dwindled with advancing age as a consequence of death. In Table X.6, the column totals fluctuate slightly as a consequence of migration. We could, however, equally well base Table X.6 on a

TABLE X.6. EDUCATIONAL EXPERIENCE FROM BIRTH TO AGE 1
AND TEN THOUSAND 19-YEAR-OLD BOYS OUTSIDE TH
BASED ON THE ADMISSION PROPORTION

| Activity \ Age | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0. Not in full-time formal education | 10,120<br>10,140 | 10,120<br>10,140 | 10,100<br>10,020 | 9,820<br>9,740 | 7,220<br>7,170 | 150<br>200 | 20<br>60 |
| 1. Nursery and primary | | | 30<br>30 | 320<br>320 | 2,910<br>2,860 | 9,960<br>9,800 | 10,030<br>9,880 |
| 2. Secondary modern: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 3. Grammar: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 4. Comprehensive: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *b*) A-level | | | | | | | |
| 5. Other normal schools: | | | | | | | |
| *a*) not A-level | | | | | | | |
| *a*) A-level | | | | | | | |
| 6. Special schools | | | | | | | 10 |
| 7. Further education n.e.s. | | | | | 10 | 20 | 40 |
| 8. Colleges of education | | | | | | | |
| 9. Universities | | | | | | | |
| Total | 10,120<br>10,140 | 10,120<br>10,140 | 10,130<br>10,050 | 10,140<br>10,060 | 10,130<br>10,040 | 10,110<br>10,020 | 10,060<br>9,980 |

coefficient matrix which ignored migrations, $H$ say, in place of $G$, just as we based Table X.5 on $D'$, which ignores deaths, rather than on $C'$. Like the $G$-matrix, the $H$-matrix would be derived from the columns of Table X.1, but the divisor would be not $p$ but $p-b$.

Ignoring this distinction, the results set out in Table X.6 can be summarized as follows. There are many routes through the educational system, and so at any age between 1 and 19 there is a distribution over activities whatever the terminal state. But depending on the terminal state the distributions are very different. Thus for the 19-year-old undergraduate the chance of being found at grammar school at 16 would have been 0.772, whereas for his contemporary outside the educational system the corresponding chance would have been 0.115.

On average, the undergraduates spent 5.2 years out of the system and 14.9 in it. Of the years spent outside, 4.8 were at the younger ages



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TEN THOUSAND 19-YEAR-OLD BOYS AT UNIVERSITY (upper rows)
STEM OF FULL-TIME FORMAL EDUCATION (lower rows)
ENGLAND AND WALES IN 1964-65

| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | | | | | | 10 | 20 | 580 | 1,090 | 1,290 | 1,400 | | 51,950 |
| 40 | 10 | 10 | 10 | | 10 | 60 | 120 | 3,990 | 7,530 | 8,900 | 9,570 | 10,000 | 87,720 |
| ,880 | 9,970 | 9,960 | 9,890 | 3,500 | 840 | 290 | | | | | | | 67,680 |
| ,830 | 9,810 | 9,790 | 9,760 | 3,100 | 430 | 70 | | | | | | | 65,680 |
| | | | | 860 | 1,230 | 1,210 | 1,190 | 690 | 180 | | | | 5,360 |
| | | | | 2,950 | 4,200 | 6,150 | 6,090 | 2,720 | 420 | 40 | | | 22,570 |
| | | | | | | | | | 10 | 20 | | | 30 |
| | | | | | | | | | 10 | | | | 10 |
| | | 10 | 80 | 4,670 | 6,720 | 7,340 | 7,630 | 6,650 | 2,730 | 10 | | | 35,840 |
| | | | 20 | 2,370 | 3,410 | 1,600 | 1,660 | 1,490 | 620 | 80 | 10 | | 11,260 |
| | | | | | | | | 970 | 4,990 | 7,610 | 3,600 | | 17,170 |
| | | | | | | | | 120 | 530 | 550 | 160 | | 1,360 |
| | | | | 70 | 130 | 170 | 210 | 210 | 90 | | | | 880 |
| | | | | 150 | 280 | 510 | 660 | 420 | 130 | 20 | | | 2,170 |
| | | | | | | | | | 120 | 240 | 140 | | 500 |
| | | | | | | | | | 20 | 30 | 10 | | 60 |
| | | | | 870 | 1,040 | 960 | 910 | 810 | 340 | | | | 4,930 |
| | | | 10 | 1,230 | 1,470 | 1,440 | 1,310 | 810 | 280 | 40 | | | 6,590 |
| | | | | | | | | 10 | 380 | 620 | 280 | | 1,290 |
| | | | | | | | | | 40 | 40 | 10 | | 90 |
| 10 | 10 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | | | | | 170 |
| 60 | 90 | 110 | 140 | 80 | 90 | 160 | 160 | 130 | 30 | | | | 1,120 |
| | | | | | | | | 50 | 70 | 40 | | | 160 |
| | | | | | | | | 360 | 490 | 370 | 250 | | 1,470 |
| | | | | | | | | | | 170 | 4,580 | 10,000 | 14,750 |
| | | | | | | | | | | 10 | 10 | | 20 |
| ,000 | 9,980 | 9,990 | 9,990 | 9,990 | 9,980 | 10,000 | 9,980 | 9,990 | 10,000 | 10,000 | 10,000 | 10,000 | 200,710 |
| ,930 | 9,910 | 9,910 | 9,940 | 9,880 | 9,890 | 9,990 | 10,000 | 10,040 | 10,100 | 10,080 | 10,020 | 10,000 | 200,120 |

and 0.4 were at the later ages, essentially between school and university. Of the years spent inside the system, 11.5 were spent below the A-level stage and 3.4 were spent at this stage or beyond.

The experience of the 19-year-olds outside the system was very different. On average, they spent 8.8 years outside the system and 11.2 years in it. Of the years spent outside, 4.8 (as for the undergraduates) were at the younger ages and 4.0, out of a theoretical maximum of 5.0, were at the later ages. Of the years spent inside the system, 10.9 were spent below the A-level stage and only 0.3 were spent at this stage or beyond.

There is nothing very surprising about these results; they simply give numerical expression to inequalities that are generally apprehended. If we take them merely as a statistical exercise, they may leave us pretty calm; but if we regard them as a spur to reducing these inequalities, their implications are formidable.

## 7. The Moral of the Figures

All the numerical examples given in this chapter measure only the time spent in different activities. Unfortunately it is impossible at present to translate them into economic language because the economic accounts of education, as we have seen, do not distinguish between the different costs of various types of education. But if we consider even approximately the expenditure involved in the kind of school work which leads on to higher education, we can see that the differences in time would be greatly magnified in terms of cost.

There is now a fast growing tendency to remain at school after the age of compulsory education. This tendency is bound to lead in the near future to a considerable reduction in the educational gap, and increasing pressure will be put on society to increase the provision of educational facilities. The force of this pressure can be estimated with some precision from the tables given above. In Table X.4, for instance, which epitomizes the educational distribution of the under-20's in 1965, the column for age 19 shows that less than 7 per cent of the young men of that age were at university; assuming a uniform distribution of ability throughout the population, as many as 50 per cent might be there if all the classes followed the pattern of the professional and managerial classes.

Such a challenge, in terms of both money and manpower, can be met only if we are prepared to accept considerable changes in educational methods. This is certainly not to deny the continuing importance of the great educational institutions that have been built up in the past, but only to suggest that a large-scale expansion of education cannot be tackled exclusively on those lines. The epidemic model described in section 4 of the next chapter is an attempt to simulate the contemporary educational explosion so as not to be caught too unprepared for it.

# XI

## PROJECTIONS AND THE PROBLEM OF CHANGING COEFFICIENTS

### 1. Methods of Projection

In economics it is generally recognized that input-output coefficients are likely to change over time because they depend on changing conditions of supply and demand. If one could explain these changing conditions, one would be in the position to build a complete Walrasian model of the productive system. Since one cannot do this, one must resort to a variety of simpler methods. These methods range from extrapolating past movements to investigating new technical developments likely to lead to change; from analysing time series to talking to technicians and businessmen. The result is a set of coefficients which one hopes will prove a fairly good approximation to the structure of the productive system in the projection year.

An exactly similar situation holds in the case of socio-demographic projections. The future numbers in the different states of the system depend partly on demographic changes and partly on changes in the structure of the system, which can be expressed as changes in the transition proportions. These proportions in turn depend partly on supply and partly on demand conditions. For instance, there may be an increasing demand to remain at school beyond the age of compulsory education; but this demand will be effective only to the extent that the supply of school places is increased.

As in the economic case, there is little possibility of explaining each demand and supply equality (transition proportion) in terms of a system of demand and supply equations, and so it is necessary to take as variables the equalities themselves. Any equation we may use to project these variables will have rather low autonomy because it ultimately depends on both demands and supplies; but it does not seem possible to do better at the present time.

Up to a point we may be able to obtain information from people directly concerned with organising and administering education. In so far as they can explain the mechanism of past changes and tell us what changes they are planning for the future, their answers will certainly help us in our extrapolations. Beyond this, however, they are not likely to know much more than we do about future demand, and it is our task to help them by building, as far as our capacities will allow it, models that bring into perspective all possible factors of change.



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If this point of view be accepted, then there only remains to work out a practicable procedure for projecting transition proportions. Such a procedure is suggested in the following sections.

## 2. Reviewing Past Changes

On the assumption that a certain amount of past data are available, the first thing to do is to see what has happened in the past and note any changes over time in individual proportions and any relationships there may be between different series of proportions. An example is set out in Table XI.1.

Table XI.1. A TIME SERIES OF TRANSITION PROPORTIONS FROM HOME TO SCHOOL : AGES 1 TO 4
England and Wales ; boys, 1960-61 to 1965-66

| | Age 1 - 2 | Age 2 - 3 | Age 3 - 4 | Age 4 - 5 |
|---|---|---|---|---|
| 1960-61 | 0.004 | 0.031 | 0.271 | 0.983 |
| 1961-62 | 0.004 | 0.031 | 0.256 | 0.969 |
| 1962-63 | 0.004 | 0.029 | 0.251 | 0.970 |
| 1963-64 | 0.003 | 0.027 | 0.260 | 0.967 |
| 1964-65 | 0.003 | 0.027 | 0.257 | 0.969 |
| 1965-66 | 0.003 | 0.033 | 0.273 | 0.966 |

Table XI.1 relates only to first admissions, that is to children who go to school for the first time. It will be noticed that there is a slight tendency for the proportions to fall as time goes on, at least up to 1964-65. For instance, the first column shows that 0.4 per cent of the boys who were aged 1 in 1960 entered school in 1961, the year of their second birthday; whereas only 0.3 per cent of the 1-year-olds of 1963 entered school in 1964. In the older ages, the drop takes place rather abruptly in 1961-62: 27.1 per cent of the 3-year-olds who were not at school in 1960 entered school in 1961, the year of their fourth birthday, whereas only 25.6 per cent of the 3-year-olds of 1961 entered school in 1962; and the corresponding proportions for the 4 to 5-year-old boys are 98.3 per cent in 1961 and 96.9 per cent in 1962.

Having noted these changes, the next step is to consider possible reasons for them. For instance, the falling tendency apparent in Table XI.1 might be due to an increasing pressure of numbers on facilities that did not keep pace with the growth of population until the end of the six-year period: the number of 1-year-olds of both sexes rose from 734,000 in 1960 to 857,200 in 1965; and the number of 4-year-olds not at school, from 481,000 in 1960 to 549,000 in 1965. On the other hand, it might be due to a greater willingness to keep young children at home as a consequence of higher incomes and greater leisure, or of some change in social *mores*.

Having examined the possible factors of change, the third step is to try to find out which of them have actually been at work. Thus in the above example none of the economic and social factors suggested should have much influence on the 4-5 ratios, since 5 is the age at which school attendance becomes compulsory. Pressure of numbers leading to a change in the regulations is a more likely explanation: for a time, institutions may

work at more than a hundred per cent of capacity, but if this becomes a regular feature one may expect to see regulations introduced which limit the pressure of attendance, such as delaying the date of first entry by a term or two.

In this connection it would be useful to develop a measure of the capacity of schools and other educational institutions. The unit would be the number of places for which provision was made. This measurement would give rise to some of the difficulties encountered in any attempt to measure plant capacity, but these difficulties are not insuperable.

As a rule, it will be impossible to reach a definite conclusion about the reasons for observed changes in the transition proportions without talking to educational administrators. But sometimes a change can confidently be attributed to a known change in circumstances. For example, in 1963 there was a change in the regulations relating to school leaving in England and Wales, which required that children whose fifteenth birthday fell in winter, who previously had been allowed to leave school at the end of the autumn term, should continue at school until the end of the following spring term. The main effect of this change of regulations is illustrated in Table XI.2.

TABLE XI.2. A TIME SERIES OF TRANSITION PROPORTIONS FOR LEAVERS FROM SECONDARY MODERN SCHOOL: AGES 14 AND 15
ENGLAND AND WALES, BOYS, 1960-61 TO 1965-66

| | Age 14 - 15 | Age 15 - 16 |
|---|---|---|
| 1960-61 | 0.793 | 0.758 |
| 1961-62 | 0.758 | 0.704 |
| 1962-63 | 0.555 | 0.724 |
| 1963-64 | 0.556 | 0.830 |
| 1964-65 | 0.519 | 0.806 |
| 1965-66 | 0.511 | 0.797 |

From the first column of Table XI.2 we can see that 75.8 per cent of the boys at secondary modern schools who were 14 in 1961 left at or around age 15 in 1962; whereas in 1963 the corresponding proportion fell to 55.5 per cent as a consequence of the change in regulations. Since we are dealing with children who leave school as soon as they can, the reduction of autumn-term leavers in one year will be made up by an increase in the spring-term leavers of the following year. This can be seen from the second column of the table, relating to those who leave school in the year in which they become 16: the proportion rises from 72.4 per cent for leavers in 1963 to 83.0 per cent for leavers in 1964.

The data on which Table XI.2 is based make it possible to calculate for any of the above years the proportions of 14-year-olds who had left school by the time they were 16. The series runs: 0.939, 0.933, 0.924, 0.916, 0.902. Thus, while there was a diminishing tendency to leave school at these ages, the sudden changes in the year-to-year proportions which appear in Table XI.2 result simply from the change in the regulations.

This example shows how a careful study of past data makes it possible to isolate once-for-all changes, which modify abruptly the structure of the



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system, from current trends which are likely to continue in the future. In either case, the more we know about the causes and effects of past changes, the better shall we be able to foresee the probable effects of future changes.

## 3. Reviewing Prospective Changes

When considering prospective changes in the transition proportions it is necessary, as it is when projecting economic input-output coefficients, to distinguish between changes that do not follow obviously from any factor that has been at work in the past and changes that are merely a continuation of past trends. An example of the former is an intended change in the school-leaving age: in Britain it is proposed to raise this age from 15 to 16 in 1972. One effect of such a change is clearly predictable: children will be required to spend their fifteenth year at school. But it is also to be expected that the change will bring about alterations in the transition proportions at later ages and in general will lead to more voluntary attendance at those ages. Here it would seem desirable to look at what happened when the leaving age was raised from 14 to 15 after the last war and at the changes in the proportions from age 15 onwards over the last twenty-five years. By these means it may be possible to make some plausible allowance for the indirect effects of the next compulsory change.

By contrast, a prospective change that follows directly from a past change is the probable acceleration in the creation of comprehensive schools. These schools, which combine secondary modern and grammar school curricula, may eventually alter the subject mix of secondary education and induce in their pupils different attitudes to further education. Since they are a recent innovation, they are still regarded with suspicion by many people: generally speaking, in those districts where a choice is possible the brighter children still tend to be sent to the grammar schools, so that the comprehensives do not get the full spectrum of abilities. In some districts, however, comprehensives have been imposed for some years now as the only form of state education, and in these cases it might soon be possible to compare their products with those of the more traditional forms of education and draw from this comparison some conclusions for the future.

Yet another recent trend is the change in the relative proportions that study science or arts subjects at university. Although the number of students reading natural sciences is rising in absolute terms, it is not rising as fast as it used to in comparison with the number of those reading arts subjects or social sciences. This phenomenon is partly attributed to a shortage of mathematics and science teachers in secondary schools, but not enough is known about it to explain it fully. It should be borne in mind, however, when making manpower projections, especially in the context of educational planning.

These examples taken from the English educational system are only a few of the many world-wide indicators of change in social attitudes to education. It is impossible to devise models that will take all prospective changes into account, let alone explain them and project their detailed consequences into the future. But it is not impossible to formulate a synthesis of their cumulative effects and to devise models that will provide some measure of the demands they are likely to place on the educational system. Two such models are outlined in the next section.

## 4. Projecting the Transition Proportions

One of the most noticeable effects of the many influences mentioned above is that as time goes on, a larger proportion of adolescents aspire to a fuller education than used to be the case. In England and Wales the school-leaving age has been 15 throughout the postwar period; but between 1950 and 1967 the proportion of each age from 15 through 19 who have elected to remain at school has doubled or more than doubled. The actual proportion varies, of course, with age and also with type of school and with region, but the general tendency is quite clear.

Apart from once-for-all changes of the kind exemplified in Table XI.2 above, and making allowance for variations induced by fluctuations in the size of vintages, these trends are not only rising but seem to be slowly accelerating. If we fitted linear trends to the underlying transitions we should almost certainly underestimate them in the short run and should inevitably overestimate them in the long run. If we fitted exponential trends, we should probably do fairly well in the short run but should, again, almost certainly overestimate the transitions in the long run and perhaps also in the medium run. In such cases an epidemic model seems likely to give fairly good results.

An example will help to clarify this idea. Let $c_{rs}$ denote the proportion of those in state $r$ in one year who are likely to be found in state $s$ in the next year. The simplest form of epidemic model is based on the assumption that the year-to-year change in this proportion, $\Delta c_{rs}$, is proportional to the product of $c_{rs}$ and $\gamma - c_{rs}$, where $\gamma$ denotes the maximum proportion of those in state $r$ who could ever be expected to be in state $s$ in the succeeding year. In most cases $\gamma < 1$. Thus:

$$\begin{aligned} \Delta c_{rs} &= \beta c_{rs}(\gamma - c_{rs}) \\ &= \beta\gamma c_{rs} - \beta c_{rs}^2 \end{aligned} \tag{XI.1}$$

where $\beta$ is the factor of proportionality and can be identified with the speed of the adjustment process. As can be seen from (XI.1), $\Delta c_{rs}$ is a quadratic function of $c_{rs}$ which, in this simple formulation, passes through the origin; and $c_{rs}$ is defined over the range $0 < c_{rs} \leqslant \gamma$. As can also be seen, $\Delta c_{rs}$ reaches a maximum when $c_{rs} = \gamma/2$; that is to say, $\Delta c_{rs}$ first rises from zero to a maximum of $\beta\gamma^2/4$ and then falls again to zero as $c_{rs}$ approaches $\gamma$. Put another way,

$$\frac{\Delta c_{rs}}{c_{rs}} = \beta\gamma - \beta c_{rs} \tag{XI.2}$$

that is to say, the proportionate change in $c_{rs}$ is a diminishing linear function of $c_{rs}$ itself.

This process is essentially a logistic process, as can be seen by substituting $\dot{c}_{rs}$ for $\Delta c_{rs}$ and writing as the differential equation:

$$\dot{c}_{rs} = \beta c_{rs}(\gamma - c_{rs}) \tag{XI.3}$$

from which it follows by integration that:

$$c_{rs} = \frac{\gamma}{1 + e^{\alpha - \beta\gamma\theta}} \tag{XI.4}$$

where $\alpha \equiv \log[(\gamma/\bar{c}_{rs}) - 1]$. In this expression, $\bar{c}_{rs}$ denotes the initial value of $c_{rs}$ at time $\theta = 0$. Thus $c_{rs}$ follows a logistic path from $\bar{c}_{rs}$ at time $\theta = 0$ towards an upper asymptote of $\gamma$. The point of inflection of this path occurs when $c_{rs} = \gamma/2$.

In economic applications, for example to the ownership of consumers' durable goods, this simple formulation is found to be too rigid to provide a good description of observed movements. It is usually necessary to recognize that the upper limit, $\gamma$, itself changes within a certain range in response to some factor which is often identified with income. A convenient method of introducing this factor has been provided by Bonus (*Die Ausbreitung des Fernsehens*, Verlag Anton Hain, Meisenheim am Glan, 1968).

Let us reformulate (XI.3) as follows:

$$\dot{c}_{rs} = \beta c_{rs}\left[1 - \frac{c_{rs}}{c^{*}_{rs}}\right] \tag{XI.5}$$

$$c^{*}_{rs} = \frac{\gamma}{1+\zeta\mu^{-\lambda}} \tag{XI.6}$$

$$\mu = \mu_0\, e^{\rho\theta}. \tag{XI.7}$$

In these equations, $\mu$ denotes real income per head; $\beta$, $\gamma$, $\zeta$, $\lambda$, $\mu_0$ and $\rho$ denote constants; and, as before, $\theta$ denotes time.

The solution of this model takes the form:

$$c_{rs} = \frac{\gamma}{1+e^{\nu-\beta\theta} + \dfrac{\beta}{\beta-\lambda\rho}\, e^{\xi-\lambda\rho\theta}} \tag{XI.8}$$

where $\nu \equiv \log \gamma\chi$, $\chi$ being the constant of integration, and $\xi \equiv \log \zeta\mu_0^{-\lambda}$. Equation (XI.8) is thus a generalization of (XI.4). It is much less rigid than (XI.4) and is capable of describing asymmetric movements from one level to another; in particular, movements which are positively skewed.

The main advantage of such projection models is that they reflect an important aspect of social change. It seems likely that the desire to reach higher rungs on the educational ladder does spread through society as society becomes richer and more homogeneous and as the advantages attributable to education become more widely recognized. Of course, if the epidemic is to take place, facilities must be expanded to make it possible; but under modern conditions this is likely to happen as a response either of public or of private enterprise to the pressure of demand. There may well be a lag in the response, but this may also be followed by a sudden increase in facilities so designed that their activity levels can be adapted to future pressures of demand. A rising demand to be educated, coupled with a rising demand for educated people and fairly rigid standards at each stage of the educational process, is likely to carry further education from a given level to a higher level of activity along some kind of sigmoid path. Even the model consisting of (XI.5) through (XI.7) leads to a comparatively simple path, but at least it avoids wholly impossible paths in which runaway movements have suddenly to be damped down completely because the whole of a particular inflow has been absorbed.

Against these favourable features there are some which are less favourable. Thus the transitions from a given state cannot, in general, all follow generalized logistics paths, since in each year they must add up to a pre-assigned number: one minus the specific mortality rate for that state. This means that there must be at least one residual transition proportion determined by subtracting the sum of the other proportions from the pre-



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assigned number. In an educational context, the transition from education to home and employment may play this role: the educational transitions are determined, and the remaining members of the age-group drop out.

Another difficulty arises as we divide up educational institutions. If the alternative to dropping out of the educational system is to remain in a particular institution, we can suppose that that institution will eventually adapt itself to an increased demand for its services. But if there are several alternatives, that is, several other institutions to which one can move within the system, and if, moreover, these alternatives can be reached from different institutions at various ages, then the observed outcome will be strongly influenced by the capacities in different parts of the system, which may not respond uniformly to the complex of pressures put upon them. If capacities do not respond, then some people will be forced back on second and lower preferences, and changes in capacities may lead to a radical rearrangement of observed transitions.

All this is only to say what is perfectly well known: a model of demand can ignore supply only if supply is perfectly elastic at the going price; and interdependent demands cannot be modelled independently. As matters stand, the epidemic model should be taken as a starting point, as a means of preventing trends from getting out of hand, and likely to give reasonably good results only under conditions of uniformly elastic supply; that is to say, under conditions in which the administrators are aware of the many pressures of demand and are successfully endeavouring to expand capacities in different parts ot the system in a uniform way to meet these demands.

## 5. Projecting the Input-Output Coefficients

If the models described in section 3 of Chapter IX are to be used for projecting cost components as well as activity levels in different branches of the educational system, then it is necessary to project the *M*-matrix as well as the *C*-matrix. The *M*-matrix is part of a general input-output coefficient matrix which contains primary as well as intermediate inputs. As in the purely economic case, the problem of projection revolves around two issues. One is product-mix: in the present context, for instance, what are 13-year-olds at academically advanced schools supposed to be learning. The other is technology: by what methods is it proposed that these things should be taught.

These issues are familiar in the projection of industrial input-output coefficients. The main difference between the two cases is one of organisation and motivation. It is generally supposed that in a relatively competitive environment there will be a constant pressure to innovate and that those who do not do so will eventually be swallowed up by those who do. It is also supposed that in a relatively monopolistic and conservative environment the pressure to innovate will come mainly from the need to meet a rising demand with limited allocations of money and of skilled manpower. If these suppositions are correct, it is likely that the practical possibilities of technical change will be more widely understood and accepted in an industrial than in an educational environment. Thus technological forecasting is likely to be more difficult in education that in industry. This is an added reason for making educational projections which will indicate the probable consequences of attempting to meet an educational explosion by traditional educational methods.

# XII

## CONCLUSIONS

### 1. Demographic Accounting in Practice

Demographic accounts and matrices provide a convenient means of arranging and presenting data on human stocks and flows. They serve two related aims: first, the integration of statistics from different fields, such as demography, education and economics; and, second, the provision of a systematic framework for the analysis of many aspects of social change, such as variations in the quantity and quality of educational output following from demographic changes and from changes in the structure of the educational system.

The numerical examples given in this report show that a start can usefully be made with existing official statistics. Once this is done, the gaps and inconsistencies in this material become apparent and it is possible to frame a realistic programme for repairing these deficiencies. Gaps and inconsistencies are to be expected at the outset, partly because it is difficult to fit together a number of separate statistical collections and partly because these collections, for instance in the fields of education and employment, concentrate more heavily on stocks than on flows and are not designed for the analysis of change.

### 2. The Advisability of a Step-by-Step Approach

One of the aims of demographic accounting is to connect all forms of social statistics, but nothing so ambitious is attempted in this report. Two aspects of man's progress through social life, the active and the passive sequences, are mentionned, but only the former is elaborated in any detail. The reason for this is twofold: in the first place, the active sequence constitutes a unity which can usefully be studied on its own; in the second place, this sequence by itself gives rise to so many problems that it seems essential not to add to them until some measure of solution has been reached.

This is a step-by-step approach, starting with limited aims and limited theories. The alternative, namely to start with general theories and all-embracing aims, is likely to be self-defeating because the complexity to be mastered is formidable and the experience on which this mastery must be founded can be acquired only gradually.

## 3. The Possibility of Developing a Standard System

If we destrict ourselves to the active sequence described in Chapters V and VI, it is clear that every country has an educational and an employment system. Of course, institutional arrangements differ from country to country; but as can be seen from the British example used in this report and the Norwegian example used by Thonstad in his study of Norwegian education, two systems with somewhat different characteristics can be subjected to very similar forms of analysis. This suggests that it is already possible to set up a simple system of standard demographic accounts which would enable the position in different countries to be compared without too much delay and at the same time would point the way to constructive developments.

Here again, excessive complexity should be avoided. It seems best to make a start by comparing the general structure of different systems, without too much insistence on the immediate reconciliation of detailed classifications and nomenclature. This conclusion stems from the economic experience: it has taken many years to reach the degree of detail embodied in the revised System of National Accounts of the United Nations; the first Standardized System, published by the OEE in 1952, was an altogether simpler affair, yet it served a useful purpose since without it the more sophisticated version would never have been formulated.

## 4. Uses and Limitations of Input-Output Models

The models presented in Chapter IX above are described and illustrated in terms of educational flows, but they could equally well be applied to the analysis of employment flows. These models make possible a wide variety of calculations which throw light on the structure of any system they are applied to. How much light depends to a large extent on the data one feeds into them. Thus it can be seen from the arithmetical illustration of Chapter IX that any change of state which follows from a given initial state is independent of the sequence of earlier changes that have led to it. In some areas this can be a serious oversimplification; for example, a student's choice of subject and performance at university may be strongly influenced by his school background; and it might therefore be desirable to classify students throughout their university career by the type of secondary school they had attended. To discover these doubtful areas a further subdivision of the categories, and hence a great deal of additional information, would be needed. The algebra of the models would not be affected, but the conclusions reached almost certainly would be.

Like all input-output models, the demographic versions proceed on rigid, fixed-choice assumptions. Up to a point this limitation can be overcome by projecting into the future any foreseeable changes in the coefficients by the methods suggested in Chapter XI. But while this combination of input-output and epidemic models can serve many uses, there remain many questions which it cannot answer. The next step will be, as it has been in economics, to develop programming models, which are better suited to dealing with problems of choice and control. However, until we have collected the data for our input-output models and have gained some understanding of the taxonomic problems they raise, there can be no question of using programming techniques, which require yet more

detailed input-output data embodying the available choices, as well as clear ideas about the objective function to be maximized of minimized.

## 5. Further Developments

While we should be careful, at this stage of demographic accounting and model-building, not to overstep the limits of what is technically and statistically possible at present, it may be useful for the organisation of future work to conclude this report with a brief survey of some of the fields of research that should be developed next. In accordance with the main lines of the report, we shall focus our attention on the application of these developments to educational policy; but many of the following remarks would, *mutatis mutandis,* hold for other aspects of the social system.

Numerous and varied writings on education have appeared in recent years. Without in any way attempting a guided tour of these studies, it is possible to summarize their subject matter under three headings: demand, supply, and planning and control.

### a) *Demand*

In most countries, education is a virtually free good produced by a virtually nationalized industry and rationed by means of educational tests and admission policies. The consumer, that is to say the student, can specify his preferences and try, by his diligence, to justify the award of his first preference. Within the limits of capacity of the system, determined by educational authorities and ultimately by the legislature, a host of institutions, joint boards and central councils decide on the distribution of the awards. Obviously, this situation is far removed from that envisaged in conventional demand analysis, where the behaviour both of producers and of consumers is governed by market values.

It is difficult to set a value on education because in many cases education is not wanted for its own sake but as a passport to a better rewarded life. Moreover, as was said above, one cannot simply buy the passport. A considerable effort is required of the student, and this effort may not be successful. Even if it is successful, a satisfactory personal outcome depends to some extent on the relationship between the product-mix of education and future economic requirements: with the present rate of technical change, there is a large element of built-in obsolescence in a highly specialized education which ends comparatively early in life. And the more remote one is from an educated milieu, the more difficult it is to see this personal planning problem in perspective.

The application of cost-benefit analysis to education can help to clarify at least the material net advantages that can be expected from it under present conditions. But if we are threatened with an educational explosion, we must try to look beyond the relative rewards of the present day, and this means that we must try to foresee in educational terms the probable future requirements of society. The question of built-in obsolescence can be tackled by considering educational options which do not exist now, in particular the possibility of promoting adaptability in youth and of meeting the need for specialized knowledge by a greater overlap of learning and earning activities.

The influence of home environment on educational aspirations,



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performance and demands also calls for much deeper study than it has so far received.

b) *Supply*

In input-output analysis there is just one average way of producing anything. In a perfectly adapted economy, for each type of output the component input vectors from which this average is formed would all be the same and would represent the most economical input structure possible, which for this reason would be adopted everywhere. In practice this situation is almost unknown: for each type of output, techniques which are still more economical are constantly being evolved and displacing existing ones. In education, the position is similar, except that there is a less pronounced tendency to think in economic terms and a more pronounced tendency to retain traditional methods.

Yet, if it is thought that the pressure on educational resources is likely to increase rather than diminish, it is particularly important to consider alternative methods and new technical possibilities. For example, if well-trained teachers are in short supply, there may be ways of using their services more effectively by substituting less skilled labour, or even machines, in some of the activities in which they are employed at present. Or, small classes in neighbouring schools could be amalgamated, at least for some courses, and taught by a single teacher. Or, again, the same results as are obtained today might be obtained more easily and more quickly with the existing input structure by introducing changes in the curricula. A study of educational techniques enables us to get a clearer picture of which input structures are the most likely to have the desired effect within some imaginable limit of cost, thought of not only in terms of money but also in terms of other scarce resources.

Having got to this point, it is necessary to recognize that a start must be made on the provision of many inputs long before their services are actually going to be used. The training of teachers and the construction of educational buildings and equipment are investment processes which take many years to complete. They are not things which a society can go and buy as it feels the need for them.

Needless to say, the financial side is very important too. Indeed, finance is the field in which the claims of one "good thing" come most evidently into conflict with the claims of other "good things". Let us leave aside the question of the claims of education against those of, say, health or defence and concentrate on conflicting claims within the educational system itself. Suppose that what we should like to see is more university graduates. Do we expect to promote this aim better by financing university students or by devoting equivalent resources to the earlier stages of education, thus engendering a greater understanding of educational values and thus a greater striving for the benefits of university education? At present the potential graduate is rather liberally treated, at least in Britain: he is abler than his fellows and his ability earns him a grant. But by spending the grant money on the improvement of secondary education we might end up with more people determined to go to a university whether they got a grant or not. In an affluent society, the possibility of this happening ought to be considered, since it would reduce to some extent the strain imposed on public funds by the educational explosion. On the other hand, it might

not happen at all, in which case there would be no escaping from the ratchet mechanism whereby the expenditures of one administration set a floor to every financial outlay of the next.

c) *Planning and control*

Planning proceeds by bringing together a knowledge to the operating characteristics of a system and a set of aims which the system is expected to serve. The combination of these two very different elements can be carried out "by hand": a group of individuals studying, formulating and discussing, and in this way finally arriving at a policy. Alternatively, an attempt can be made to formalize the policy-making process and to incorporate some, if not all, of it in a model. To do this we need a model of a different kind from those described above. This more ambitious type of model must contain the following features: *i*) the operating characteristics of the system must be specified in such a way that the choices open to the operator are made clear; *ii*) this specification will already imply some restrictions on what is possible, and any further restrictions to be observed as a matter of policy must be stated explicitly; and *iii*) the utility function in terms of which the performance of the system is to be judged must be defined.

If these features can be elaborated, then a variety of programming methods are available for working out how the system should be operated so as to maximize utility. The optimum solution provides a standard against which the required inputs can be valued, and these notional values can be compared with actual costs. One should beware, however, of relying exclusively on the answers given by a model; indeed, the more sophisticated the model, the more sensitive will it be to unrealistic features or assumptions which we may unwittingly have embodied in its formulation, so that some of its answers may be positively misleading. In other words, we should review the outcome of any model "by hand" before we accept it as a basis for policy. With this precaution, programming models can be a powerful aid to policy-making.

Once a particular policy has been decided on with the help of some such model, other problems arise: it is one thing to decide on a policy, it is another to ensure that the policy is viable. To make it viable, a whole complex of administrative arrangements, financial inducements, information drives and so on may be required. It is necessary to devise a socially acceptable set of controls for dealing with these factors before a policy can be embarked on with any confidence.

In principle, it is possible to extend a programming model to incorporate these problems too. But we are still far from such an achievement. Our immediate task is to bring into being a co-ordinated body of statistics which will enable us to turn what can now only be a theoretical exercise into a practical tool for social policy.

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| | | | Age 0 | Age 1 | Age 2 | | | Age 3 | | | Age 4 | | | Age 5 | | | Age 6 | | | Age 7 | | | Age 8 | | | Age 9 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 2 |
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| Age 0 | 0 not in full-time formal education | 1 | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 1 | 0 not in full-time formal education | 2 | 0.998 | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 2 | 0 not in full-time formal education | 3 | 0.994 | 0.994 | 1.000 | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 nursery & primary | 4 | 0.003 | 0.003 | | 1.000 | | | | | | | | | | | | | | | | | | | | | | |
| | 2 special schools | 5 | 0.001 | 0.001 | | | 1.000 | | | | | | | | | | | | | | | | | | | | | |
| Age 3 | 0 not in full-time formal education | 6 | 0.946 | 0.945 | 0.931 | | | 1.000 | | | | | | | | | | | | | | | | | | | | |
| | 1 nursery & primary | 7 | 0.019 | 0.038 | 0.037 | 1.000 | | | 1.000 | | | | | | | | | | | | | | | | | | | |
| | 2 special schools | 8 | 0.001 | 0.001 | | | 1.000 | | | 1.000 | | | | | | | | | | | | | | | | | | |
| Age 4 | 0 not in full-time formal education | 9 | 0.301 | 0.316 | 0.311 | | | 0.341 | | | 1.000 | | | | | | | | | | | | | | | | | |
| | 1 nursery & primary | 10 | 0.131 | 0.237 | 0.236 | 1.000 | | 0.256 | 1.000 | | | 1.000 | | | | | | | | | | | | | | | | |
| | 2 special schools | 11 | 0.001 | 0.001 | 0.001 | | 1.000 | 0.001 | | 1.000 | | | 1.000 | | | | | | | | | | | | | | | |
| Age 5 | 0 not in full-time formal education | 12 | 0.021 | 0.021 | 0.021 | | | 0.021 | | | 0.030 | | | 1.000 | | | | | | | | | | | | | | |
| | 1 nursery & primary | 13 | 0.951 | 0.920 | 0.923 | 1.000 | | 0.934 | 1.000 | | 0.967 | 1.000 | | | 1.000 | | | | | | | | | | | | | |
| | 2 special schools | 14 | 0.003 | 0.003 | 0.003 | | 1.000 | 0.001 | | 1.000 | 0.001 | | 1.000 | | | 1.000 | | | | | | | | | | | | |
| Age 6 | 0 not in full-time formal education | 15 | 0.009 | 0.009 | 0.009 | | | 0.009 | | | 0.013 | | | 0.618 | | | 1.000 | | | | | | | | | | | |
| | 1 nursery & primary | 16 | 0.961 | 0.960 | 0.956 | 1.000 | | 0.985 | 1.000 | | 0.981 | 1.000 | | 0.519 | 1.000 | | | 1.000 | | | | | | | | | | |
| | 2 special schools | 17 | 0.004 | 0.004 | 0.004 | | 1.000 | 0.004 | | 1.000 | 0.004 | | 1.000 | 0.044 | | 1.000 | | | 1.000 | | | | | | | | | |
| Age 7 | 0 not in full-time formal education | 18 | 0.001 | 0.001 | 0.001 | | | 0.001 | | | 0.010 | | | 0.316 | | | 0.358 | | | 1.000 | | | | | | | | |
| | 1 nursery & primary | 19 | 0.960 | 0.959 | 0.953 | 0.998 | | 0.956 | 0.998 | | 0.981 | 0.998 | | 0.516 | 0.998 | | | 0.999 | | | 1.000 | | | | | | | |
| | 2 special schools | 20 | 0.039 | 0.039 | 0.039 | 0.001 | 1.000 | 0.039 | 0.001 | 1.000 | 0.009 | 0.001 | 1.000 | 0.167 | 0.001 | 1.000 | 0.641 | 0.001 | 1.000 | | | 1.000 | | | | | | |
| Age 8 | 0 not in full-time formal education | 21 | 0.004 | 0.004 | 0.004 | | | 0.004 | | | 0.005 | | | 0.164 | | | 0.391 | | | 0.517 | | | 1.000 | | | | | |
| | 1 nursery & primary | 22 | 0.960 | 0.959 | 0.953 | 0.998 | | 0.956 | 0.998 | | 0.981 | 0.998 | | 0.548 | 0.998 | | 0.578 | 0.998 | | 0.103 | 0.999 | | | 1.000 | | | | |
| | 2 special schools | 23 | 0.040 | 0.040 | 0.039 | 0.001 | 1.000 | 0.039 | 0.001 | 1.000 | 0.011 | 0.001 | 1.000 | 0.287 | 0.001 | 1.000 | 0.530 | 0.001 | 1.000 | 0.379 | | 1.000 | | | 1.000 | | | |
| Age 9 | 0 not in full-time formal education | 24 | | | | | | | | | | | | | | | | | | | | | | | | 1.000 | | |
| | 1 nursery & primary | 25 | 0.957 | 0.956 | 0.950 | 0.994 | | 0.958 | 0.994 | | 0.978 | 0.994 | | 0.544 | 0.994 | | 0.698 | 0.995 | | 0.103 | 0.996 | | | 0.997 | | | 1.000 | |
| | 3 grammar | 26 | 0.001 | 0.001 | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | | 1.000 |
| | 6 special schools | 27 | 0.043 | 0.043 | 0.043 | 0.005 | 1.000 | 0.043 | 0.005 | 1.000 | 0.019 | 0.005 | 1.000 | 0.451 | 0.005 | 1.000 | 0.911 | 0.005 | 1.000 | 0.897 | 0.005 | 1.000 | 1.000 | 0.001 | 1.000 | 1.000 | 0.001 | |
| Age 10 | 0 not in full-time formal education | 28 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 nursery & primary | 29 | 0.951 | 0.951 | 0.944 | 0.989 | | 0.935 | 0.989 | | 0.933 | 0.989 | | 0.543 | 0.989 | | 0.678 | 0.989 | | 0.103 | 0.990 | | | 0.991 | | | 0.995 | |
| | 3 grammar | 30 | 0.004 | 0.004 | 0.004 | 0.004 | | 0.004 | 0.004 | | 0.004 | 0.004 | | 0.001 | 0.004 | | | 0.004 | | | 0.004 | | | 0.004 | | | 0.003 | 1.000 |
| | 5 other schools | 31 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 special schools | 32 | 0.017 | 0.017 | 0.017 | 0.005 | 1.000 | 0.017 | 0.005 | 1.000 | 0.010 | 0.005 | 1.000 | 0.453 | 0.005 | 1.000 | 0.711 | 0.005 | 1.000 | 0.897 | 0.004 | 1.000 | 1.000 | 0.004 | 1.000 | 1.000 | 0.002 | |
| Age 11 | 1 nursery & primary | 33 | 0.309 | 0.315 | 0.314 | 0.321 | | 0.316 | 0.381 | | 0.315 | 0.321 | | 0.176 | 0.321 | | 0.025 | 0.321 | | 0.035 | 0.321 | | | 0.321 | | | 0.321 | |
| | 2 secondary modern | 34 | 0.369 | 0.376 | 0.375 | 0.381 | | 0.376 | 0.381 | | 0.375 | 0.381 | | 0.209 | 0.381 | | 0.030 | 0.381 | | 0.040 | 0.381 | | | 0.381 | | | 0.383 | |
| | 3 grammar | 35 | 0.146 | 0.147 | 0.148 | 0.150 | | 0.148 | 0.150 | | 0.147 | 0.150 | | 0.082 | 0.150 | | 0.012 | 0.150 | | 0.016 | 0.150 | | | 0.150 | | | 0.150 | 1.000 |
| | 4 comprehensive | 36 | 0.090 | 0.091 | 0.091 | 0.093 | | 0.091 | 0.093 | | 0.091 | 0.093 | | 0.040 | 0.093 | | 0.004 | 0.093 | | 0.008 | 0.093 | | | 0.093 | | | 0.093 | |
| | 5 other schools | 37 | 0.044 | 0.043 | 0.042 | 0.043 | | 0.042 | 0.043 | | 0.043 | 0.043 | | 0.038 | 0.043 | | 0.005 | 0.043 | | 0.003 | 0.043 | | | 0.043 | | | 0.043 | |
| | 6 special schools | 38 | 0.013 | 0.013 | 0.013 | 0.005 | 1.000 | 0.013 | 0.005 | 1.000 | 0.010 | 0.005 | 1.000 | 0.453 | 0.005 | 1.000 | 0.911 | 0.005 | 1.000 | 0.893 | 0.004 | 1.000 | 1.000 | 0.004 | 1.000 | 1.000 | 0.002 | |
| Age 12 | 0 not in full-time formal education | 39 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 nursery & primary | 40 | 0.015 | 0.014 | 0.014 | 0.014 | | 0.014 | 0.014 | | 0.014 | 0.014 | | 0.014 | 0.014 | | 0.001 | 0.014 | | 0.003 | 0.014 | | | 0.014 | | | 0.014 | |
| | 2 secondary modern | 41 | 0.534 | 0.534 | 0.534 | 0.545 | | 0.533 | 0.545 | | 0.534 | 0.545 | | 0.199 | 0.545 | | 0.043 | 0.545 | | 0.056 | 0.545 | | | 0.544 | | | 0.549 | |
| | 3 grammar | 42 | 0.208 | 0.212 | 0.213 | 0.216 | | 0.213 | 0.216 | | 0.212 | 0.216 | | 0.119 | 0.216 | | 0.017 | 0.216 | | 0.021 | 0.216 | | | 0.216 | | | 0.216 | 1.000 |
| | 4 comprehensive | 43 | 0.116 | 0.118 | 0.119 | 0.120 | | 0.119 | 0.120 | | 0.118 | 0.120 | | 0.064 | 0.120 | | 0.009 | 0.120 | | 0.013 | 0.121 | | | 0.121 | | | 0.121 | |
| | 5 other schools | 44 | 0.082 | 0.083 | 0.083 | 0.083 | | 0.084 | 0.085 | | 0.083 | 0.085 | | 0.047 | 0.085 | | 0.007 | 0.085 | | 0.009 | 0.085 | | | 0.085 | | | 0.085 | |
| | 6 special schools | 45 | 0.018 | 0.018 | 0.018 | 0.006 | 1.000 | 0.018 | 0.006 | 1.000 | 0.011 | 0.006 | 1.000 | 0.454 | 0.006 | 1.000 | 0.911 | 0.006 | 1.000 | 0.892 | 0.005 | 1.000 | 1.000 | 0.005 | 1.000 | 1.000 | 0.003 | |
| Age 13 | 0 not in full-time formal education | 46 | 0.002 | 0.002 | 0.001 | 0.001 | 0.019 | 0.002 | 0.001 | 0.019 | 0.002 | 0.001 | 0.019 | 0.009 | 0.001 | 0.019 | 0.018 | 0.001 | 0.019 | 0.017 | 0.001 | 0.019 | 0.019 | 0.001 | 0.019 | 0.019 | 0.001 | |
| | 1 nursery & primary | 47 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 secondary modern | 48 | 0.504 | 0.504 | 0.504 | 0.523 | | 0.516 | 0.523 | | 0.515 | 0.523 | | 0.288 | 0.523 | | 0.041 | 0.524 | | 0.054 | 0.524 | | | 0.525 | | | 0.526 | |
| | 3 grammar | 49 | 0.234 | 0.239 | 0.240 | 0.243 | | 0.240 | 0.243 | | 0.239 | 0.243 | | 0.134 | 0.243 | | 0.019 | 0.244 | | 0.025 | 0.244 | | | 0.244 | | | 0.244 | |
| | 4 comprehensive | 50 | 0.151 | 0.154 | 0.155 | 0.157 | | 0.155 | 0.157 | | 0.155 | 0.157 | | 0.086 | 0.157 | | 0.012 | 0.157 | | 0.016 | 0.157 | | | 0.158 | | | 0.158 | |
| | 5 other schools | 51 | 0.063 | 0.064 | 0.064 | 0.065 | | 0.064 | 0.065 | | 0.064 | 0.065 | | 0.036 | 0.065 | | 0.005 | 0.065 | | 0.007 | 0.065 | | | 0.065 | | | 0.066 | |
| | 6 special schools | 52 | 0.018 | 0.018 | 0.018 | 0.006 | 1.000 | 0.018 | 0.006 | 0.981 | 0.011 | 0.006 | 0.981 | 0.455 | 0.006 | 0.981 | 0.903 | 0.006 | 0.981 | 0.890 | 0.005 | 0.981 | 0.981 | 0.005 | 0.981 | 0.981 | 0.003 | |
| Age 14 | 0 not in full-time formal education | 53 | 0.313 | 0.319 | 0.331 | 0.334 | 0.135 | 0.331 | 0.334 | 0.135 | 0.331 | 0.334 | 0.135 | 0.294 | 0.334 | 0.135 | 0.150 | 0.334 | 0.135 | 0.155 | 0.334 | 0.135 | 0.135 | 0.335 | 0.135 | 0.135 | 0.335 | 0.010 |
| | 1 secondary modern : a non 'A' level | 54 | 0.241 | 0.249 | 0.248 | 0.251 | | 0.248 | 0.251 | | 0.247 | 0.251 | | 0.138 | 0.251 | | 0.010 | 0.251 | | 0.014 | 0.251 | | | 0.253 | | | 0.253 | |
| | 3 grammar : a non 'A' level | 55 | 0.209 | 0.213 | 0.213 | 0.216 | | 0.213 | 0.216 | | 0.213 | 0.216 | | 0.119 | 0.216 | | 0.017 | 0.216 | | 0.021 | 0.216 | | | 0.217 | | | 0.217 | 0.997 |
| | b 'A' level | 56 | 0.021 | 0.021 | 0.021 | 0.021 | | 0.021 | 0.021 | | 0.021 | 0.021 | | 0.011 | 0.021 | | 0.001 | 0.021 | | 0.001 | 0.021 | | | 0.021 | | | 0.021 | 0.003 |
| | 4 comprehensive : a non 'A' level | 57 | 0.105 | 0.107 | 0.107 | 0.109 | | 0.107 | 0.109 | | 0.107 | 0.109 | | 0.060 | 0.109 | | 0.009 | 0.109 | | 0.011 | 0.109 | | | 0.109 | | | 0.109 | |
| Age 15 | b 'A' level | 58 | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 |
| | 5 other schools : a non 'A' level | 59 | 0.038 | 0.039 | 0.039 | 0.040 | | 0.039 | 0.040 | | 0.039 | 0.040 | | 0.021 | 0.040 | | 0.003 | 0.040 | | 0.004 | 0.040 | | | 0.040 | | | 0.040 | |
| | b 'A' level | 60 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 special schools : a non 'A' level | 61 | 0.015 | 0.015 | 0.015 | 0.005 | 0.847 | 0.015 | 0.005 | 0.847 | 0.018 | 0.005 | 0.847 | 0.384 | 0.005 | 0.847 | 0.781 | 0.005 | 0.847 | 0.759 | 0.004 | 0.847 | 0.847 | 0.004 | 0.847 | 0.847 | 0.002 | |
| | b 'A' level | 62 | 0.018 | 0.018 | 0.018 | 0.018 | 0.019 | 0.018 | 0.019 | 0.019 | 0.018 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.001 |
| | 7 further education not elsewhere specified | 63 | 0.018 | 0.018 | 0.018 | 0.018 | 0.019 | 0.018 | 0.019 | 0.019 | 0.018 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.019 | 0.001 |
| Age 16 | 0 not in full-time formal education | 64 | 0.631 | 0.644 | 0.646 | 0.644 | 0.749 | 0.647 | 0.644 | 0.749 | 0.647 | 0.644 | 0.749 | 0.693 | 0.644 | 0.749 | 0.741 | 0.644 | 0.749 | 0.739 | 0.647 | 0.749 | 0.749 | 0.647 | 0.749 | 0.749 | 0.648 | 0.181 |
| | 2 secondary modern : a non 'A' level | 65 | 0.045 | 0.046 | 0.046 | 0.047 | | 0.046 | 0.047 | | 0.046 | 0.047 | | 0.026 | 0.047 | | 0.004 | 0.047 | | 0.005 | 0.049 | | | 0.049 | | | 0.049 | |
| | b 'A' level | 66 | 0.002 | 0.002 | 0.002 | 0.002 | | 0.002 | 0.002 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | |
| | 3 grammar : a non 'A' level | 67 | 0.086 | 0.088 | 0.088 | 0.090 | | 0.089 | 0.090 | | 0.088 | 0.090 | | 0.049 | 0.090 | | 0.007 | 0.090 | | 0.009 | 0.090 | | | 0.090 | | | 0.090 | 0.346 |
| | b 'A' level | 68 | 0.103 | 0.105 | 0.105 | 0.107 | | 0.105 | 0.107 | | 0.105 | 0.107 | | 0.059 | 0.107 | | 0.008 | 0.107 | | 0.011 | 0.107 | | | 0.107 | | | 0.107 | 0.423 |
| Age 16 | 4 comprehensive : a non 'A' level | 69 | 0.035 | 0.035 | 0.035 | 0.036 | | 0.035 | 0.036 | | 0.035 | 0.036 | | 0.020 | 0.036 | | 0.003 | 0.036 | | 0.004 | 0.036 | | | 0.036 | | | 0.036 | |
| | b 'A' level | 70 | 0.013 | 0.013 | 0.013 | 0.013 | | 0.013 | 0.013 | | 0.013 | 0.013 | | 0.007 | 0.013 | | 0.001 | 0.013 | | 0.001 | 0.013 | | | 0.013 | | | 0.013 | 0.005 |
| | 5 other schools : a non 'A' level | 71 | 0.012 | 0.013 | 0.013 | 0.013 | | 0.013 | 0.013 | | 0.013 | 0.013 | | 0.007 | 0.013 | | 0.001 | 0.013 | | 0.001 | 0.013 | | | 0.013 | | | 0.013 | |
| | b 'A' level | 72 | 0.004 | 0.004 | 0.004 | 0.004 | | 0.004 | 0.004 | | 0.004 | 0.004 | | 0.002 | 0.004 | | | 0.004 | | | 0.004 | | | 0.004 | | | 0.004 | |
| | 6 special schools : a non 'A' level | 73 | 0.004 | 0.004 | 0.004 | 0.001 | 0.236 | 0.004 | 0.001 | 0.236 | 0.005 | 0.001 | 0.236 | 0.001 | 0.001 | 0.236 | 0.001 | 0.001 | 0.236 | 0.001 | 0.001 | 0.236 | 0.236 | 0.001 | 0.236 | 0.236 | 0.001 | |
| | b 'A' level | 74 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 further education not elsewhere specified | 75 | 0.036 | 0.033 | 0.033 | 0.039 | 0.014 | 0.033 | 0.039 | 0.014 | 0.033 | 0.039 | 0.014 | 0.017 | 0.039 | 0.014 | 0.016 | 0.039 | 0.014 | 0.019 | 0.039 | 0.014 | 0.014 | 0.039 | 0.014 | 0.014 | 0.039 | 0.002 |
| | 0 not in full-time formal education | 76 | 0.954 | 0.949 | 0.951 | 0.951 | 0.962 | 0.951 | 0.951 | 0.962 | 0.953 | 0.951 | 0.962 | 0.959 | 0.951 | 0.962 | 0.947 | 0.951 | 0.962 | 0.941 | 0.952 | 0.962 | 0.962 | 0.952 | 0.962 | 0.962 | 0.952 | 0.333 |
| | 2 secondary modern : a non 'A' level | 77 | 0.005 | 0.005 | 0.005 | 0.005 | | 0.005 | 0.005 | | 0.005 | 0.005 | | 0.003 | 0.005 | | | 0.005 | | 0.001 | 0.005 | | | 0.005 | | | 0.005 | |
| | b 'A' level | 78 | 0.001 | 0.001 | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.002 | | | 0.001 | | | 0.001 | | | 0.001 | |
| | 3 grammar : a non 'A' level | 79 | 0.011 | 0.011 | 0.011 | 0.011 | | 0.011 | 0.011 | | 0.011 | 0.011 | | 0.006 | 0.011 | | 0.001 | 0.011 | | 0.001 | 0.011 | | | 0.011 | | | 0.011 | 0.001 |
| | b 'A' level | 80 | 0.130 | 0.131 | 0.133 | 0.138 | | 0.133 | 0.138 | | 0.133 | 0.135 | | 0.076 | 0.135 | | 0.011 | 0.135 | | 0.014 | 0.135 | | | 0.135 | | | 0.135 | 0.512 |
| Age 17 | comprehensive : a non 'A' level | 81 | 0.006 | 0.006 | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.003 | 0.006 | | 0.001 | 0.006 | | 0.001 | 0.006 | | | 0.006 | | | 0.006 | |
| | b 'A' level | 82 | 0.020 | 0.021 | 0.021 | 0.021 | | 0.021 | 0.021 | | 0.021 | 0.021 | | 0.012 | 0.021 | | 0.001 | 0.021 | | 0.001 | 0.021 | | | 0.021 | | | 0.021 | |
| | 5 other schools : a non 'A' level | 83 | 0.001 | 0.001 | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | |
| | b 'A' level | 84 | 0.005 | 0.005 | 0.005 | 0.005 | | 0.005 | 0.005 | | 0.005 | 0.005 | | 0.003 | 0.005 | | | 0.005 | | 0.001 | 0.005 | | | 0.005 | | | 0.005 | |
| | 6 special schools : a non 'A' level | 85 | | | | | 0.010 | | | 0.010 | | | 0.010 | 0.009 | | 0.010 | 0.018 | | 0.010 | 0.018 | | 0.010 | 0.010 | | 0.010 | 0.010 | | |
| | 7 further education not elsewhere specified | 86 | 0.035 | 0.035 | 0.036 | 0.036 | 0.018 | 0.036 | 0.036 | 0.018 | 0.036 | 0.036 | 0.018 | 0.018 | 0.036 | 0.018 | 0.019 | 0.036 | 0.018 | 0.018 | 0.036 | 0.018 | 0.018 | 0.036 | 0.018 | 0.018 | 0.036 | 0.001 |
| | 8 colleges of education | 87 | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 |
| | 9 universities | 88 | 0.001 | 0.001 | [illegible] | 80 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | 0.001 |
| | 0 not in full-time formal education | 89 | 0.931 | 0.[illegible] | 940 | 0.940 | 0.938 | 0.941 | 0.940 | 0.938 | 0.941 | 0.940 | 0.938 | 0.903 | 0.940 | 0.938 | 0.961 | 0.941 | 0.938 | 0.964 | 0.941 | 0.938 | 0.938 | 0.941 | 0.938 | 0.938 | 0.941 | 0.556 |
| | 2 secondary modern : a non 'A' level | 90 | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 |
| | b 'A' level | 91 | 0.001 | 0.001 | 0.0 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | 0.001 |
| | 3 grammar : a non 'A' level | 92 | 0.001 | 0.[illegible] | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | 0.001 |
| | b 'A' level | 93 | 0.055 | 0.[illegible] | 0.056 | 0.057 | | 0.056 | 0.057 | | 0.056 | 0.057 | | 0.031 | 0.057 | | 0.005 | 0.057 | | 0.006 | 0.057 | | | 0.057 | | | 0.057 | 0.213 |
| Age 18 | 4 comprehensive : a non 'A' level | 94 | 0.001 | 0.001 | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | 0.004 |
| | b 'A' level | 95 | 0.011 | 0.011 | 0.011 | 0.011 | | 0.011 | 0.011 | | 0.011 | 0.011 | | 0.006 | 0.011 | | 0.001 | 0.011 | | 0.001 | 0.011 | | | 0.011 | | | 0.011 | 0.042 |
| | 5 other schools : a non 'A' level | 96 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | b 'A' level | 97 | 0.001 | 0.001 | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | 0.001 | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | | | 0.001 | |
| | 7 further education not elsewhere specified | 98 | 0.036 | 0.037 | 0.037 | 0.037 | 0.010 | 0.037 | 0.037 | 0.010 | 0.037 | 0.037 | 0.010 | 0.019 | 0.037 | 0.010 | 0.021 | 0.037 | 0.010 | 0.021 | 0.037 | 0.010 | 0.010 | 0.037 | 0.010 | 0.010 | 0.037 | 0.001 |
| | 8 colleges of education | 99 | 0.006 | 0.006 | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.003 | 0.006 | | 0.001 | 0.006 | | 0.001 | 0.006 | | | 0.006 | | | 0.006 | 0.001 |
| | 9 universities | 100 | 0.035 | 0.036 | 0.036 | 0.036 | | 0.036 | 0.036 | | 0.036 | 0.036 | | 0.020 | 0.036 | | 0.003 | 0.036 | | 0.004 | 0.036 | | | 0.036 | | | 0.036 | 0.001 |
| | 0 not in full-time formal education | 101 | 0.950 | 0.947 | 0.950 | 0.951 | 0.948 | 0.951 | 0.951 | 0.948 | 0.951 | 0.951 | 0.948 | 0.915 | 0.951 | 0.948 | 0.940 | 0.951 | 0.948 | 0.958 | 0.952 | 0.948 | 0.948 | 0.952 | 0.948 | 0.948 | 0.952 | 0.556 |
| | 3 grammar : a non 'A' level | 102 | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 |
| | b 'A' level | 103 | 0.006 | 0.006 | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.006 | 0.006 | | 0.004 | 0.006 | | 0.001 | 0.006 | | 0.001 | 0.006 | | | 0.006 | | | 0.006 | 0.002 |
| Age 19 | 4 comprehensive : a non 'A' level | 104 | 0.001 | 0.001 | 0.001 | 0.003 | | 0.001 | 0.003 | | 0.001 | 0.003 | | 0.001 | 0.003 | | | 0.003 | | | 0.003 | | | 0.003 | | | 0.003 | 0.001 |
| | b 'A' level | 105 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 other schools : a non 'A' level | 106 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | b 'A' level | 107 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 further education not elsewhere specified | 108 | 0.030 | 0.030 | 0.030 | 0.031 | 0.018 | 0.031 | 0.031 | 0.018 | 0.030 | 0.031 | 0.018 | [illegible] | 0.031 | 0.018 | 0.019 | 0.031 | 0.018 | 0.029 | 0.031 | 0.018 | 0.018 | 0.031 | 0.018 | 0.018 | 0.031 | 0.001 |
| | 8 colleges of education | 109 | 0.013 | 0.013 | 0.013 | 0.014 | 0.003 | 0.013 | 0.014 | 0.003 | 0.013 | 0.014 | 0.003 | [illegible] | 0.014 | 0.003 | 0.004 | 0.014 | 0.003 | 0.004 | 0.014 | 0.003 | 0.003 | 0.014 | 0.003 | 0.003 | 0.014 | 0.001 |
| | 9 universities | 110 | 0.068 | 0.068 | 0.068 | 0.068 | 0.009 | 0.068 | 0.068 | 0.009 | 0.067 | 0.068 | 0.009 | [illegible] | 0.068 | 0.009 | 0.013 | 0.068 | 0.009 | 0.015 | 0.068 | 0.009 | 0.009 | 0.068 | 0.009 | 0.009 | 0.068 | 0.002 |

TABLE X.3. THE MATRIX MULTIPLIER $(I-C')^{-1}$, DERIVED FROM TABLE X.2

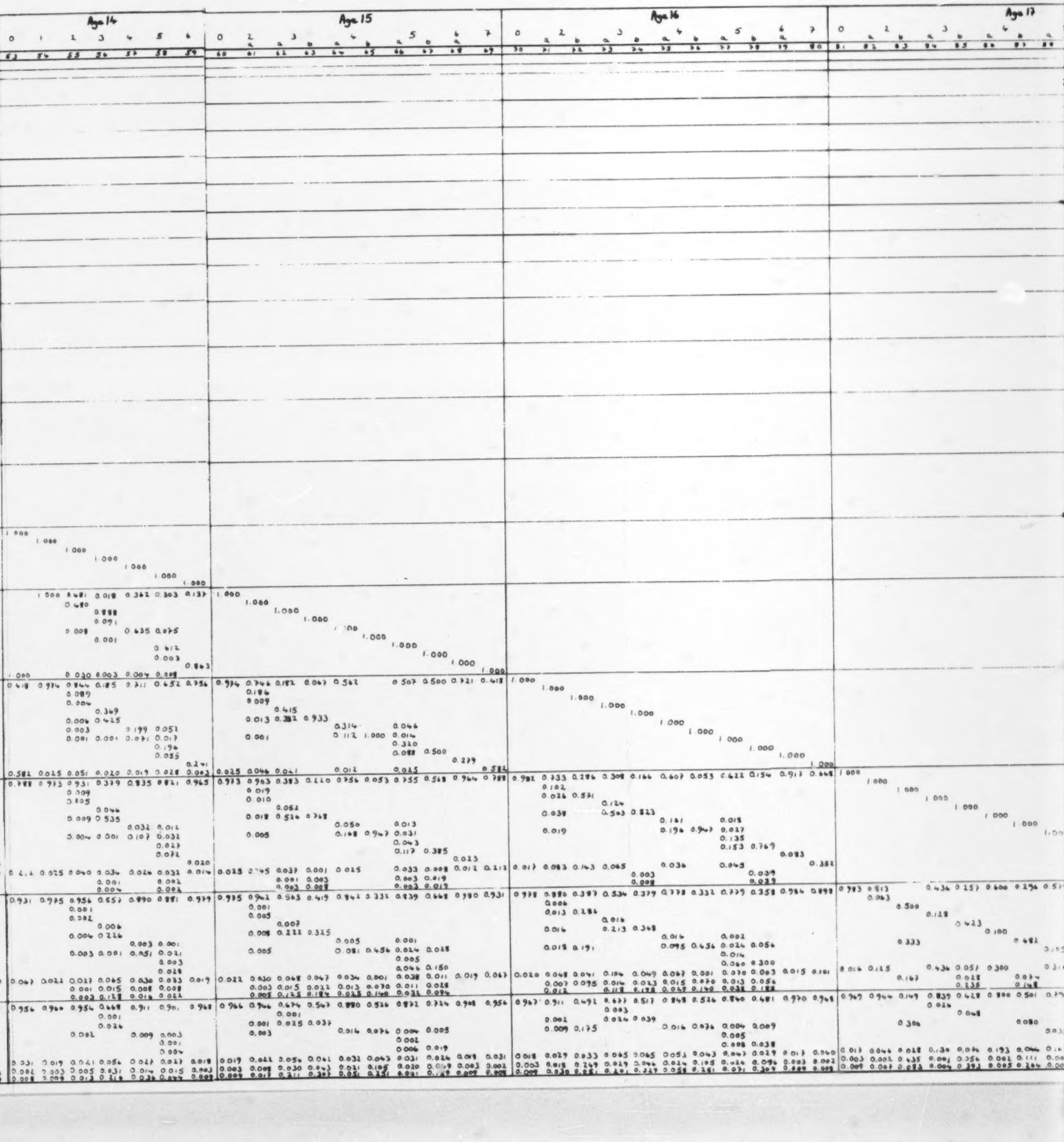
England and Wales, boys

120

130

TABLEAU X.3. MULTIPLICATEUR MATRICIEL $(I-C')^{-1}$, DÉRIVÉ DU TABLEAU X.2

Angleterre et Pays de Galles, garçons



1

| | Age 17 | | | | | | | | | | Age 18 | | | | | | | | | | Age 19 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 |
| | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 0.783 | 0.813 | 0.436 | 0.257 | 0.600 | 0.256 | 0.537 | 0.144 | 1.000 | 0.341 | 0.500 | 1.000 | 0.800 | 0.291 | 0.619 | 0.313 | 0.715 | 0.035 | 0.049 | 1.000 | 0.500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 0.063 | 0.500 | 0.128 | 0.423 | 0.100 | 0.481 | 0.033 | 0.105 | 0.190 | | | | 0.200 | 0.113 | 0.183 | 0.323 | 0.125 | | | | | | | | | | | | | |
| | 0.016 | 0.125 | 0.436 | 0.057 | 0.300 | 0.316 | 0.158 | 0.163 | 0.028 | 0.034 | 0.033 | 0.138 | 0.148 | 0.210 | 0.500 | | | | | | | | | | | | | | | |
| | 0.969 | 0.944 | 0.149 | 0.839 | 0.628 | 0.900 | 0.501 | 0.290 | 0.405 | 0.935 | 0.908 | 0.512 | 0.935 | 1.000 | 0.800 | 0.291 | 0.619 | 0.313 | 0.715 | 0.035 | 0.049 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | | 0.026 | 0.048 | | | | | | | | | | 0.200 | 0.113 | | | | | | | | | | | | | | |
| | | | | | | | | 0.304 | | | 0.080 | | 0.012 | | 0.500 | | 0.183 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | 0.323 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | 0.125 | | | | | | | | | | | |
| | 0.019 | 0.046 | 0.028 | 0.130 | 0.034 | 0.193 | 0.044 | 0.169 | 0.034 | 0.013 | 0.083 | 0.006 | 0.013 | | 0.133 | 1.000 | 0.083 | 0.667 | 0.063 | 0.185 | | | | | | | | | | |
| | 0.003 | 0.001 | 0.435 | 0.001 | 0.354 | 0.001 | 0.111 | 0.001 | 0.124 | 0.003 | 0.001 | 0.001 | 0.003 | | 0.500 | | 0.083 | | 0.125 | | 0.254 | | | | | | | | | |
| | 0.009 | 0.001 | 0.433 | 0.001 | 0.191 | 0.003 | 0.249 | 0.205 | 0.338 | 0.022 | 0.001 | 0.001 | 0.009 | | | | 0.458 | | 0.338 | | | | | | | | | | | |

TABLE X.1. AN EDUCATION

| | | | Outside world flows 1964 | Age 0 | Age 1 | Age 2 | | | Age 3 | | | Age 4 | | | Age 5 | | | Age 6 | | | Age 7 | | | Age 8 | | | Age 9 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 0 | 0 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 3 |
| | | | 1964 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 26 |
| Outside world births & immigrations 1965 | | | | 443.2 | | 0.5 | | | 0.3 | | | -0.7 | | | -0.9 | | | -1.9 | | | -1.9 | | | -0.9 | | | 0.1 | |
| Age 0 | 0 | not in full-time formal education | 1 | 0.9 | | 442.2 | | | | | | | | | | | | | | | | | | | | | | |
| Age 1 | 0 | not in full-time formal education | 2 | 1.4 | | | 436.9 | 1.1 | 0.2 | | | | | | | | | | | | | | | | | | | |
| Age 2 | 0 | not in full-time formal education | 3 | 0.8 | | | | | | 428.0 | 11.4 | 0.1 | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 4 | | | | | | | | 1.3 | | | | | | | | | | | | | | | | | |
| | 4 | special schools | 5 | | | | | | | | | 0.2 | | | | | | | | | | | | | | | | |
| Age 3 | 0 | not in full-time formal education | 6 | 0.3 | | | | | | | | 191.8 | 41.1 | 0.3 | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 7 | | | | | | | | | | 1.6 | | | | | | | | | | | | | | | |
| | 4 | special schools | 8 | | | | | | | | | | | 0.2 | | | | | | | | | | | | | | |
| Age 4 | 0 | not in full-time formal education | 9 | 0.2 | | | | | | | | | | | 8.5 | 132.4 | 0.6 | | | | | | | | | | | |
| | 1 | nursery & primary | 10 | | | | | | | | | | | | | 112.1 | | | | | | | | | | | | |
| | 4 | special schools | 11 | | | | | | | | | | | | | | 0.4 | | | | | | | | | | | |
| Age 5 | 0 | not in full-time formal education | 12 | 0.2 | | | | | | | | | | | | | | 5.8 | 4.3 | 0.4 | | | | | | | | |
| | 1 | nursery & primary | 13 | | | | | | | | | | | | | | | | 365.9 | | | | | | | | | |
| | 4 | special schools | 14 | | | | | | | | | | | | | | | | | 1.0 | | | | | | | | |
| Age 6 | 0 | not in full-time formal education | 15 | 0.2 | | | | | | | | | | | | | | | | | 2.5 | | 0.8 | | | | | |
| | 1 | nursery & primary | 16 | | | | | | | | | | | | | | | | | | | 365.0 | 0.2 | | | | | |
| | 4 | special schools | 17 | | | | | | | | | | | | | | | | | | | | 1.5 | | | | | |
| Age 7 | 0 | not in full-time formal education | 18 | 0.2 | | | | | | | | | | | | | | | | | | | | 1.5 | 0.3 | 1.1 | | |
| | 1 | nursery & primary | 19 | | | | | | | | | | | | | | | | | | | | | | 354.9 | | | |
| | 4 | special schools | 20 | | | | | | | | | | | | | | | | | | | | | | | 1.6 | | |
| Age 8 | 0 | not in full-time formal education | 21 | 0.2 | | | | | | | | | | | | | | | | | | | | | | | | 341.3 0.2 |
| | 1 | nursery & primary | 22 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | special schools | 23 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 9 | 0 | not in full-time formal education | 24 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 25 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 26 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | special schools | 27 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 10 | 0 | not in full-time formal education | 28 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 29 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 30 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 31 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 32 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 11 | 1 | nursery & primary | 33 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 34 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 35 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 36 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 37 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 38 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 12 | 0 | not in full-time formal education | 39 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 40 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 41 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 42 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 43 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 44 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 45 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 13 | 0 | not in full-time formal education | 46 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 47 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 48 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 49 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 50 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 51 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 52 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 14 | 0 | not in full-time formal education | 53 | 0.2 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 54 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 55 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 56 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 57 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 58 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 59 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 15 | 0 | not in full-time formal education | 60 | 0.2 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : a non 'A' level | 61 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : a non 'A' level | 62 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 63 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : a non 'A' level | 64 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 65 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : a non 'A' level | 66 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 67 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : a non 'A' level | 68 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | 69 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 16 | 0 | not in full-time formal education | 70 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : a non 'A' level | 71 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 72 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : a non 'A' level | 73 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 74 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : a non 'A' level | 75 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 76 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : a non 'A' level | 77 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 78 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : a non 'A' level | 79 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | 80 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 17 | 0 | not in full-time formal education | 81 | 0.4 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : a non 'A' level | 82 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 83 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : a non 'A' level | 84 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 85 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : a non 'A' level | 86 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 87 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : a non 'A' level | 88 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 89 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : a non 'A' level | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | 91 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | 92 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | 93 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 18 | 0 | not in full-time formal education | 94 | 0.4 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : a non 'A' level | 95 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 96 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : a non 'A' level | 97 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 98 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : a non 'A' level | 99 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 100 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : a non 'A' level | 101 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 102 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | 103 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | 104 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | 105 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 19 | 0 | not in full-time formal education | 106 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : a non 'A' level | 107 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 108 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : b 'A' level | 109 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : a non 'A' level | 110 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | b 'A' level | 111 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | 112 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | 113 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | 114 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Total flows 1965 | | | | 443.2 | 442.2 | 437.2 | 1.1 | 0.2 | 440.3 | 12.7 | 0.3 | 292.1 | 113.7 | 0.5 | 8.5 | 383.7 | 1.0 | 3.8 | 368.8 | 1.6 | 2.5 | 363.2 | 2.5 | 1.5 | 354.2 | 3.5 | 341.4 | 0.2 |

1964

CATIONAL MATRIX FOR ACTUAL RESIDENTS : AGES 0 THROUGH 19
England and Wales, boys, 1964-1965

TABLEAU X.1. MATRICE DES AC

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1965

| Age 9 | | | Age 10 | | | | | Age 11 | | | | | | Age 12 | | | | | | | Age 13 | | | | | | | Age 14 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 6 | 0 | 1 | 3 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 2 |
| 48 | 46 | 47 | 29 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 |
| 0.1 | | | | 0.2 | | | | | 0.1 | | | | | | -0.2 | -0.1 | -0.1 | -0.1 | | | | -0.2 | | | | | | | 0.2 | | | | | | | 0.9 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 341.3 | 0.2 | 0.3<br>0.8<br>3.8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | 325.0 | 1.0<br>0.1 | 0.1 | 0.2<br>0.6<br>4.0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | 106.6 | 124.9 | 49.6<br>0.9 | 24.1 | 22.9<br>0.1 | 0.5<br>4.8 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | 8.8 | 58.1<br>130.3 | 23.1<br>50.3 | 16.1<br>18.6 | 5.4<br>27.3 | 0.3<br>5.2 | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | 3.0 | 0.3<br>180.5 | 5.8<br>71.0 | 3.7<br>25.9 | 0.7 | 3.7 | 32.2 | 0.2<br>5.3 | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.7<br>0.6<br>0.1 | 0.1<br>181.7 | 3.0<br>17.0<br>0.1 | 3.3<br>15.7<br>4.1 | 33.4 | 5.3 | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.1<br>91.5<br>1.4<br>9.4<br>12.1<br>0.7 | 91.3 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 341.4 | 0.2 | 4.5 | | 325.2 | 1.1 | 0.1 | 4.8 | 106.6 | 124.9 | 49.5 | 24.1 | 22.9 | 5.3 | 8.8 | 195.1 | 72.3 | 34.6 | 32.6 | 5.5 | | 3.0 | 180.6 | 37.5 | 33.3 | 37.9 | 5.5 | | 1.4 | 0.1 | 181.9 | 20.1 | 33.1 | 33.4 | 5.2 | 116.1 | 91.3 |

LEAU X.1. MATRICE DES ACTIVITÉS ÉDUCATIVES DES RÉSIDENTS DE FAIT, DE L'ÂGE 0 A L'AGE 19 ANS RÉVOLUS
Angleterre et Pays de Galles, garçons, 1964-1965

1965

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| Age 14 | Age 15 | Age 16 | Age 17 |
|---|---|---|---|
| 3 4 5 6 | 0 2 3 4 5 6 7 | 0 1 2 3 4 5 6 7 | 0 1 2 3 4 5 6 7 8 |
| 2 | 0.9 0.5 | 1.5 0.1 | 0.1 |
| 1.7 3.0 3.3 | | | |
| 37.0 | | | |
| 25.7 | | | |
| 0.1 4.1 33.4 5.2 | | | |
| | 0.1 | | |
| | 91.5 9.3 | | |
| | 1.4 71.1 7.3 1.5 | | |
| | 9.4 16.5 0.1 | | |
| | 12.1 3.0 24.4 0.1 4.4 0.3 | | |
| | 0.9 | | |
| | 3.4 | | |
| | 5.6 | | |
| | 0.2 | | |
| | 0.1 | | |
| | 0.3 | | |
| | | 124.3 3.2 | |
| | | 68.8 17.1 0.8 1.2 0.1 4.2 | |
| | | 13.2 30.2 27.8 1.5 | |
| | | 0.5 7.0 | |
| | | 9.5 5.3 1.9 0.3 | |
| | | 0.1 | |
| | | 4.4 1.3 0.4 9.1 2.5 0.9 | |
| | | 0.1 0.1 | |
| | | 3.1 1.2 | |
| | | 8.1 2.1 | |
| | | | 256.4 4.4 |
| | | | 11.5 1.6 0.4 0.6 0.3 1.3 |
| | | | 0.3 0.4 0.1 |
| | | | 9.9 4.0 16.3 2.1 |
| | | | 5.9 39.3 0.1 |
| | | | 3.4 0.9 1.1 0.2 |
| | | | 0.1 1.8 |
| | | | 6.9 0.2 0.3 1.5 1.7 0.5 |
| | | | 0.6 2.0 0.1 0.1 |
| | | | 1.1 6.3 |
| | | | 11.6 |
| 31.9 38.1 33.1 33.4 5.2 | 116.1 91.3 71.1 7.3 21.0 0.1 24.4 0.1 4.4 10.1 | 240.5 17.1 0.8 30.2 36.0 6.6 2.5 9.1 2.6 1.2 17.0 | 307.5 1.6 0.8 4.0 44.1 1.1 3.5 1.5 3.7 0.1 14.9 0.2 |

A L'AGE 19 ANS RÉVOLUS



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Age 17 Age 18 Age 19

Totaux généraux 1964

127

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TABLE X.2

1965

| | | | Age 0 | Age 1 | Age 2 | | | Age 3 | | | Age 4 | | | Age 5 | | | Age 6 | | | Age 7 | | | Age 8 | | | Age 9 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0 | 0 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 4 | 0 | 1 | 3 |
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| Age 0 | 0 | not in full-time formal education | 1 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 1 | 0 | not in full-time formal education | 2 | 0.980 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 2 | 0 | not in full-time formal education | 3 | | 0.994 | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 4 | | 0.003 | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 5 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 3 | 0 | not in full-time formal education | 6 | | | 0.93 | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 7 | | | 0.067 | .000 | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 8 | | | | .000 | | | | | | | | | | | | | | | | | | | | | |
| Age 4 | 0 | not in full-time formal education | 9 | | | | | 0.141 | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 10 | | | | | 0.854 | .000 | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 11 | | | | | 0.001 | | .000 | | | | | | | | | | | | | | | | | | |
| Age 5 | 0 | not in full-time formal education | 12 | | | | | | | | 0.039 | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 13 | | | | | | | | 0.961 | .000 | | | | | | | | | | | | | | | | |
| | 6 | special schools | 14 | | | | | | | | 0.004 | | .000 | | | | | | | | | | | | | | | |
| Age 6 | 0 | not in full-time formal education | 15 | | | | | | | | | | | 0.418 | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 16 | | | | | | | | | | | 0.514 | 0.999 | | | | | | | | | | | | | |
| | 6 | special schools | 17 | | | | | | | | | | | 0.068 | | .000 | | | | | | | | | | | | |
| Age 7 | 0 | not in full-time formal education | 18 | | | | | | | | | | | | | | 0.758 | | | | | | | | | | | |
| | 1 | nursery & primary | 19 | | | | | | | | | | | | | | | 0.999 | | | | | | | | | | |
| | 6 | special schools | 20 | | | | | | | | | | | | | | 0.242 | 0.001 | .000 | | | | | | | | | |
| Age 8 | 0 | not in full-time formal education | 21 | | | | | | | | | | | | | | | | | 0.513 | | | | | | | | |
| | 1 | nursery & primary | 22 | | | | | | | | | | | | | | | | | 0.103 | 0.999 | | | | | | | |
| | 6 | special schools | 23 | | | | | | | | | | | | | | | | | 0.384 | | .000 | | | | | | |
| Age 9 | 0 | not in full-time formal education | 24 | | | | | | | | | | | | | | | | | | | | | 0.996 | | | | |
| | 1 | nursery & primary | 25 | | | | | | | | | | | | | | | | | | | | | 0.001 | | | | |
| | 3 | grammar | 26 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 27 | | | | | | | | | | | | | | | | | | | | .000 | 0.003 | .000 | | | |
| Age 10 | 0 | not in full-time formal education | 28 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 29 | | | | | | | | | | | | | | | | | | | | | | | | 0.994 | |
| | 3 | grammar | 30 | | | | | | | | | | | | | | | | | | | | | | | | 0.003 | .000 |
| | 5 | other schools | 31 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 32 | | | | | | | | | | | | | | | | | | | | | | | .000 | 0.001 | |
| Age 11 | 1 | nursery & primary | 33 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 34 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 35 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 36 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 37 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 38 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 12 | 0 | not in full-time formal education | 39 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 40 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 41 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 42 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 43 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 44 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 45 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 13 | 0 | not in full-time formal education | 46 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 47 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 48 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 49 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 50 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 51 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 52 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 14 | 0 | not in full-time formal education | 53 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | nursery & primary | 54 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | secondary modern | 55 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar | 56 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive | 57 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools | 58 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools | 59 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 15 | 0 | not in full-time formal education | 60 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : | a non 'A' level | 61 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 62 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : | a non 'A' level | 63 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 64 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : | a non 'A' level | 65 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 66 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : | a non 'A' level | 67 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 68 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : | a non 'A' level | 69 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | | 70 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 16 | 0 | not in full-time formal education | | 71 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : | a non 'A' level | 72 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 73 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : | a non 'A' level | 74 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 75 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : | a non 'A' level | 76 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 77 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : | a non 'A' level | 78 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 79 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : | a non 'A' level | 80 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | | 81 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 17 | 0 | not in full-time formal education | | 82 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : | a non 'A' level | 83 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 84 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : | a non 'A' level | 85 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 86 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : | a non 'A' level | 87 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 88 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : | a non 'A' level | 89 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 90 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | special schools : | a non 'A' level | 91 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | | 92 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | | 93 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | | 94 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 18 | 0 | not in full-time formal education | | 95 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | secondary modern : | a non 'A' level | 96 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 97 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : | a non 'A' level | 98 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 99 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : | a non 'A' level | 100 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 101 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : | a non 'A' level | 102 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 103 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | | 104 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | | 105 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | | 106 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 19 | 0 | not in full-time formal education | | 107 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | grammar : | a non 'A' level | 108 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 109 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | comprehensive : | b 'A' level | 110 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | other schools : | a non 'A' level | 111 | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | b 'A' level | 112 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | further education not elsewhere specified | | 113 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | colleges of education | | 114 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | universities | | 115 | | | | | | | | | | | | | | | | | | | | | | | | |

TRIX OF TRANSITION PROPORTIONS, C', DERIVED FROM TABLE X.1
England and Wales, boys

128 136

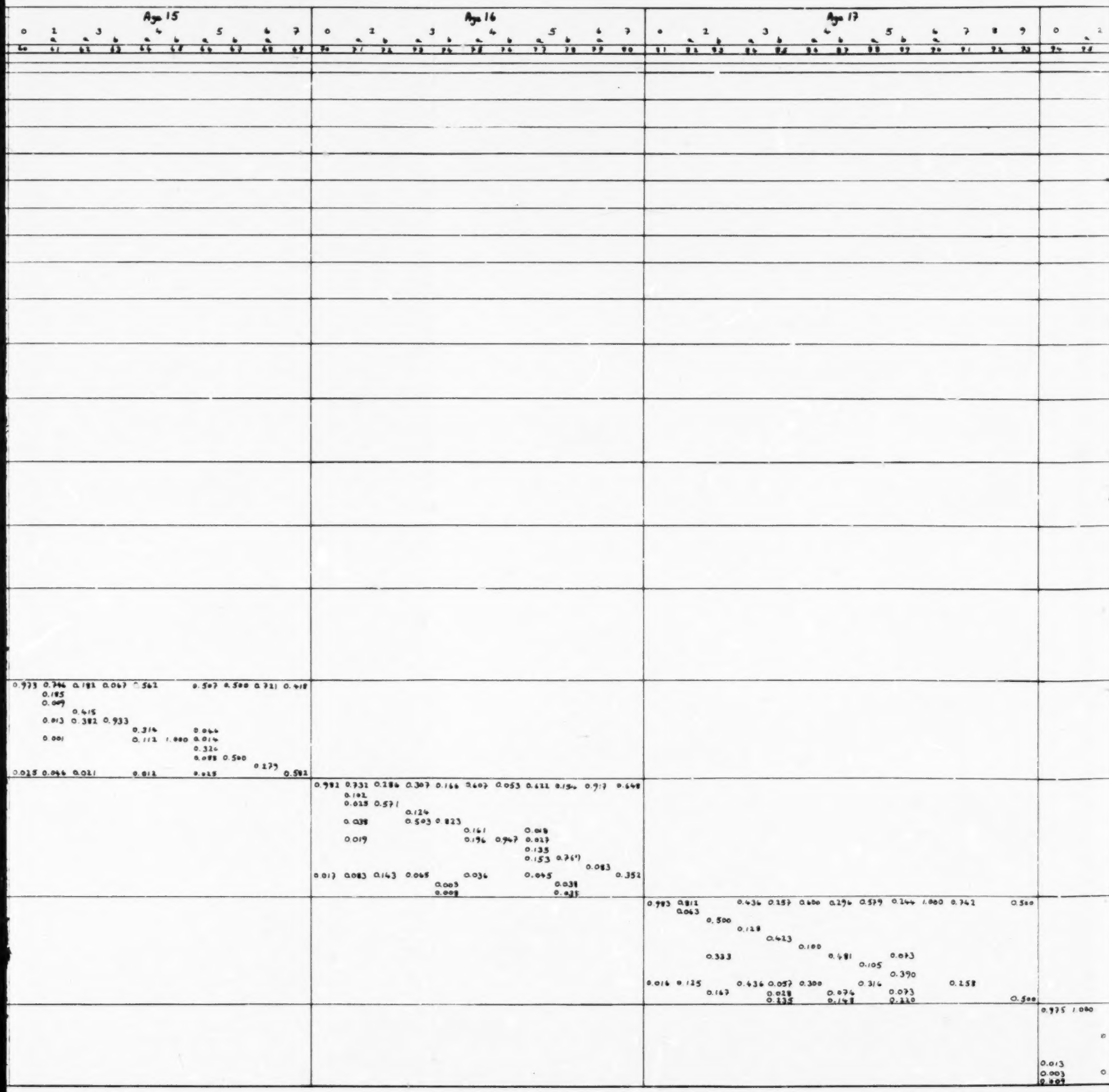
TABLEAU X.2. MATRICE DES COE

1964

| Age 10 | | | Age 11 | | | | | | Age 12 | | | | | | | Age 13 | | | | | | | Age 14 | | | | | | | Age 15 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 0 | 2 | 3 | 4 | 5 | 6 | |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 |
| 24 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 85 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 48 | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 93 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 49 | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.082 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.510 | 0.999 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.204 | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.149 | | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.050 | | | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0.003 | | | | | 1.000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | 0.600 | 0.341 | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | 0.659 | 0.979 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | 0.010 | 1.000 | | 1.000 | 0.019 | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | 0.099 | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | 0.400 | | | | | | 0.881 | 1.000 | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | 1.000 | | | | | | 0.016 | 0.019 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | 0.032 | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | 0.981 | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | 0.968 | | 1.000 | | | 0.003 | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | 0.018 | | 1.000 | 0.107 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | 0.874 | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | 0.981 | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | 1.000 | 0.481 | 0.017 | 0.362 | 0.303 | 0.137 | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | 0.480 | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | 0.888 | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | 0.091 | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | 0.008 | | 0.634 | 0.035 | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.611 | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.003 | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.863 | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | 1.000 | | 0.029 | 0.001 | 0.004 | 0.008 | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.973 | 0.746 | 0.181 | 0.047 | 0.562 | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.185 | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.009 | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.615 | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.013 | 0.381 | 0.933 | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.314 | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.001 | | | 0.112 | 1.000 | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.025 | 0.046 | 0.071 | | 0.013 | |

2. MATRICE DES COEFFICIENTS DE TRANSITION, C', DÉRIVÉE DU TABLEAU X.1

Angleterre et Pays de Galles, garçons



137

130

| Age 17 | | |